Math 76 - Homework 1

Due Friday, July 7

1. Use MATLAB to solve the linear system

$$\begin{aligned} 3x_1 + 2x_2 + x_3 &= 11 \\ 2x_1 + 3x_2 + x_3 &= 13 \\ x_1 + x_2 + 4x_3 &= 12 \end{aligned}$$

   using two different methods.

2. We say a $n \times n$ matrix $A = (a_{ij})_{n\times n}$ is symmetric when for all $1 \leq i, j \leq n$,

$$a_{ij} = a_{ji}.$$

   Write a MATLAB function that will test if a matrix is symmetric or not. The function will return the number "1" if the matrix is symmetric and "0" if it is not.

3. The *Fibonacci sequence* is defined by

$$F_n = \begin{cases} 1, & n = 1; \\ 1, & n = 2; \\ F_{n-1} + F_{n-2}, & n \geq 3. \end{cases}$$

   (a) Write a script which calculates F_m. Use a for loop.

   (b) Write a script which finds N^* such that $F_{N^*} < m$ and $F_{N^*+1} \geq m$. Use a while loop.

   (c) Write a script which finds the sum of the first m *Fibonacci numbers* $F_n, 1 \leq n \leq m$, for which F_n is divisible by either 2 or 5, that is

$$\sum_{n=1}^{m} \begin{cases} F_n, & \text{if divisible by 2 or 5;} \\ 0, & \text{otherwise.} \end{cases}$$

   You should find the MATLAB built-in function **mod** helpful.

   (The m above is a input value given by the user.)

4. Consider the second-order ordinary differential equation

$$y''(t) + y(t) = 0.$$

   (a) Solve this ODE analytically using initial conditions $y(0) = 2$ and $y'(0) = 1$.

(b) Rewrite this ODE as a system of two first-order differential equations.

(c) Write a function in MATLAB that represents the system of first-order ODEs.

(d) Using the same initial conditions, solve the system numerically over the interval $[0, 10]$. How many steps were used? Generate a plot with the numerical and analytic solutions to the second-order ODE versus time.

(e) Compare the analytic solution to the numerical solution. Generate plots of the absolute and relative errors. What is the error at $t = 10$?

5. Recall that a first-order system of linear differential equations with constant coefficients may be expressed in matrix notation as

$$\frac{dY}{dt} = AY,$$

where $Y(t)$ is a vector-valued function and A is a square matrix (with constant coefficients). Moreover, if λ_1 is an eigenvalue for A (i.e. $\det(A - \lambda_1 I) = 0$) with associated eigenvector V_1 (i.e. $AV_1 = \lambda_1 V_1$), then

$$Y(t) = e^{\lambda_1 t} V_1$$

is a solution of the equation. We shall now use MATLAB to compute the eigenvalues and eigenvectors of a given square matrix A, and therefore the analytic solutions of the equation. Consider the system of equations

$$\begin{cases} \frac{dx}{dt} = -x - 4y, \\ \frac{dy}{dt} = 3x - 2y. \end{cases}$$

(a) Use MATLAB to determine the eigenvalues and eigenvectors of the associated matrix.

(b) Use (a) to find two linearly-independent solutions and the general solution of the equation.

(c) Use (b) to find the solution satisfying the initial conditions $x(0) = 1$ and $y(0) = -1$.

(d) Compare the problem with the last one. Does the method offered here work for a general ODE problem? Why or why not?

6. Consider the dataset

Year	Price
1	8.3
2	8.3
3	9.5
4	10.5
6	8
7	8.5
8	8.4
9	8.6
10	9.1

Notice that a price for year 5 is missing.

(a) Linearly interpolate between years 4 and 6 to approximate the price for year 5. Plot the data with the approximation.

(b) Fit a polynomial to the entire dataset to find the corresponding price for year 5. Decide which order polynomial fits best. Plot the data with the model.

(c) Explain the differences in these approaches.