Math 8 Winter 2009 — Midterm 2 Review Problems

Solutions will be available at the course webpage `http://www.math.dartmouth.edu/~m8w09/` by Thursday, February 19th.

1 Evaluate $\int x\cos^2(3x)\,dx$

2 Evaluate $\int e^{2x}\sin x\,dx$.

3 Could you in principle compute $\int x^{10^{10}}e^x\,dx$, and if so, how?

4 Evaluate $\int \sin^3(x)\cos^4(x)\,dx$.

5 Evaluate $\int \sec^4(x)\tan^4(x)\,dx$.

6 What substitution would you use to evaluate $\int x^3\sqrt{16+x^2}\,dx$?

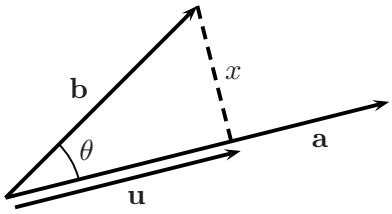
7 Evaluate $\int \frac{dx}{(9-x^2)^{3/2}}\,dx$.

8 Is the angle between the vectors $\mathbf{a} = \langle 3,-1,2\rangle$ and $\mathbf{b} = \langle 2,2,4\rangle$ acute, obtuse, or right?

9 Find the area of the parallelogram whose vertices are $(-1,2,0)$, $(0,4,2)$, $(2,1,-2)$, and $(3,3,0)$.

10 If $\mathbf{a}$ and $\mathbf{b}$ are both nonzero vectors and $\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}\times\mathbf{b}|$, what can you say about the relationship between $\mathbf{a}$ and $\mathbf{b}$?

11 Consider the vectors $\mathbf{a} = \langle 4,1\rangle$ and $\mathbf{b} = \langle 2,2\rangle$, shown below. Compute $\cos\theta$, $\mathbf{u}$, and the length x.



Note: you should not leave unevaluated trigonometric functions in your answer.

12 Find the equation of the plane which passes through the point $(2, -3, 1)$ and contains the line

$$x = 3t - 2, \quad y = t + 3, \quad z = 5t - 3 \ .$$

13 Find the line of intersection of the planes $x + y + z = 12$ and $2x + 3y + z = 2$.

14 Compute the position vector for a particle which passes through the origin at time $t = 0$ and has velocity vector

$$\mathbf{r}(t) = 2t\,\mathbf{i} + \sin t\,\mathbf{j} + \cos t\,\mathbf{k}.$$

15 Show that if a particle moves at constant speed, then its velocity and acceleration vectors are orthogonal. Note that this does *not* mean that the velocity is 0! (Hint: consider the derivative of $\mathbf{v} \cdot \mathbf{v}$.)

16 Consider the curve defined by

$$\mathbf{r}(t) = \langle 4\sin ct, 3ct, 4\cos ct \rangle \ .$$

What value of c makes the arc length of the space curve traced by $\mathbf{r}(t)$, $0 \leq t \leq 1$, equal to 10?