PRACTICE PROBLEMS

These problems were considered for inclusion in the final exam, but were rejected for various reasons. Some are too difficult, some too long, etc. You should not assume that the final exam will resemble these problems with regard to content, difficulty, length, etc. However they are useful to practice on.

(a) Find the equation of the tangent plane to the surface given by $\ln(x+z) - yz = 3$ at the point $(0, -3, 1)$.

(b) Let

$$\mathbf{r}(t) = \langle t^2 + 5,\ \tfrac{4}{3}t^{\frac{3}{2}},\ t - 7\rangle,$$

be a curve in 3-space from $t = 0$ to $t = 1$.

a. What is the distance between $\mathbf{r}(0)$ and $\mathbf{r}(1)$?

b. What is the length of the curve?

(c) Find the directional derivative of

$$f(x, y, z) = \frac{1}{x^2 + y^2 + z^2}$$

at $(-1, 0, 1)$ in the direction from $(-1, 0, 1)$ to $(1, 2, 2)$. In which direction is the function increasing at the greatest rate at $(-1, 0, 1)$?

(d) Find the (absolute) maximum and minimum values of $f(x, y, z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 30$,

(e) Find all points on the surface given by $3z = x^2 + xy$ where the normal line to the surface is parallel to the line given by $x = 2t + 1\ y = 4t - 1\ z = -3t + 7$.

(f) Suppose that f is a differentiable function from R^2 to R, that $f(1, 2) = 5$ and that the tangent plane to the graph of f at the point $(1, 2, 5)$ is given by

$$z - 5 = 4(x - 1) + 9(y - 2).$$

Write down the equation of the tangent line to the level curve $f(x, y) = 5$ at (1,2).

(g) Evaluate $\int_0^{\frac{1}{2}} \sqrt{1 - x^2}\, dx$

(h) Find the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{n^3(x-1)^n}{3^{2n}}.$$

(i) Find the second degree Taylor polynomial for $f(x) = \tan(x)$ about $x = \frac{\pi}{4}$.

(j) Integrate

$$\int \frac{(\log x)\sin(\log(x^3))}{x}\,dx$$

(k) Consider a parallelogram spanned by $\mathbf{v} = \langle 1, 2\rangle$ and $\mathbf{u}$, where $\mathbf{u}$ is a unit vector.
(a) For what value or values of $\mathbf{u}$ is the parallelogram degenerate; ie, not actually a parallelogram?
(b) For what value or values of $\mathbf{u}$ is the area of the parallelogram maximized? What is the area in this case?
(c) Suppose $\mathbf{u} = \langle 3/5, 4/5\rangle$. What is the area of the parallelogram?

(l) Let $f(x, y) = xy+x+y+1$. Find the absolute maximum and minimum of this function on the region $x^2 + y^2 \leq 2, x \geq 0, y \geq 0$.

(m) Suppose that f is a differentiable function and that $\nabla(f)(2, 3) = \langle 2, 4\rangle$. If $\mathbf{u}$ is a unit vector that makes an angle of $\frac{\pi}{3}$ with $\nabla f(2, 3)$, find the directional derivative $D_{\mathbf{u}}(2, 3)$.

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NAME : ______________________________

SECTION : ______________________________

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Problem	Points	Score
Total	0	