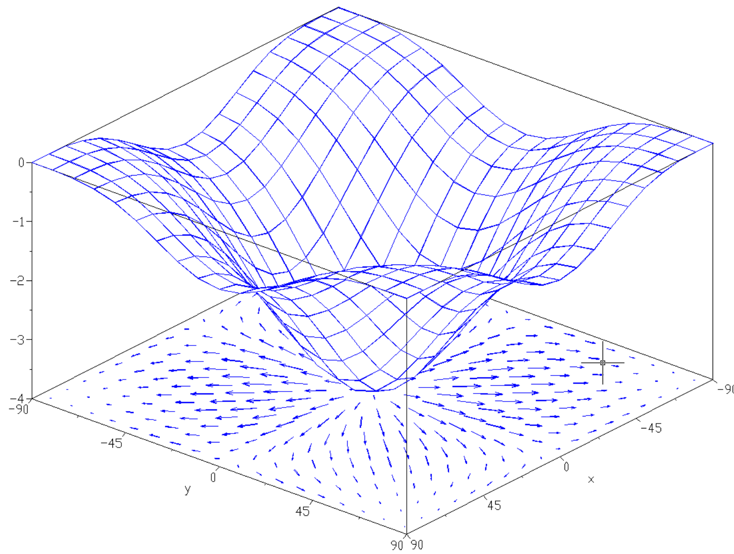
1. Gradients

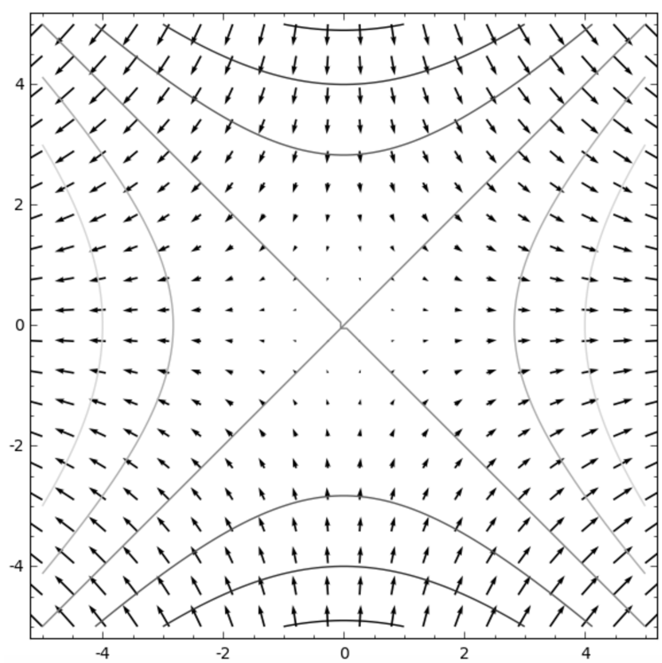
1.1. Functions of two variables. For a function of two variables $f(x, y)$, the gradient $\nabla f = \langle f_x, f_y \rangle$ is a vector valued function of x and y. At a point (a, b), the gradient $\langle f_x(a, b), f_y(a, b) \rangle$ is a vector in the xy-plane that points in the direction of the greatest increase for $f(x, y)$.



[1]

1.2. Examples: Functions of two variables.

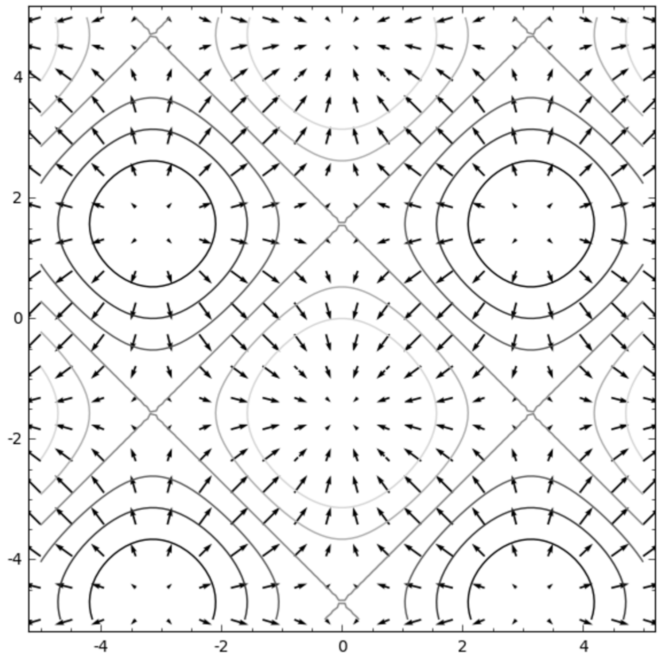
$$(1)\ f(x, y) = x^2 - y^2$$



$$\nabla f = \langle f_x, f_y \rangle = \langle 2x, -2y \rangle$$

[1]Image from `https://commons.wikimedia.org/wiki/File:Gradient99.png`

(2) $f(x, y) = \cos(x) + \sin(y)$

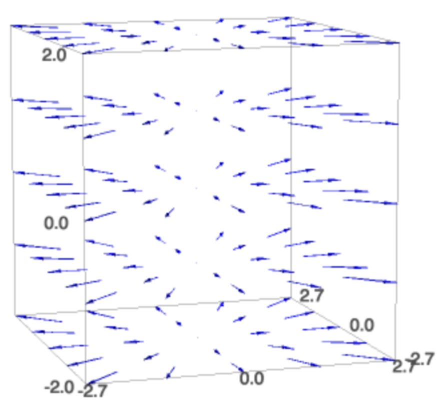


$\nabla f = \langle f_x, f_y \rangle = \langle -\sin(x), \cos(y) \rangle$

1.3. **Functions of three variables.**

For a function of 3-variables $f(x, y, z)$, the gradient $\nabla f = \langle f_x, f_y, f_z \rangle$ is a vector valued function of x, y and z. At a point (a, b, c), the gradient $\langle f_x(a, b, c), f_y(a, b, c), f_z(a, b, c) \rangle$ is a vector in $\mathbb{R}^3$ that points in the direction of the greatest increase for $f(x, y, z)$.

Example: $f(x, y, z) = x^2 + y^2 + 1$



$\nabla f = \langle f_x, f_y, f_z \rangle = \langle 2x, 2y, 0 \rangle$