Math 8, Fall 2019, Exam I Practice

1. Use the definition of limit to show that $\lim_{n\to\infty} \dfrac{n^2+1}{n^2} = 1$.

2. (a) Find the sum: $\displaystyle\sum_{k=4}^{\infty} \frac{2\cdot 3^k}{5^{2k+1}} =$

   (b) Determine whether the series converges: $\displaystyle\sum_{k=1}^{\infty} \frac{k^3 2^k}{3^{k-1}}$

   (c) Find the limit: $\lim_{n\to\infty} \dfrac{\ln(n)}{n} =$

3. (a) Compute $T_{10}(x)$, the 10th degree Taylor Polynomial, of $\sin(x)\cos(x)$ centered at $x = 0$.

   (b) Find the radius of convergence of the corresponding Taylor series using Taylor's Inequality.

   Taylor's Inequality: If $T_n(x)$ is the degree n Taylor polynomial of f centered at $x = a$, and $B_{n+1} \geq |f^{(n+1)}(w)|$ for all w between a and x, then $|f(x) - T_n(x)|$ is at most $\dfrac{B_{n+1}}{(n+1)!}|x-a|^{n+1}$.

4. Between noon and 2 PM a car is traveling with velocity at t hours past noon equal to $65 - 2t$ miles per hour.

   (a) Over a very short time interval of length Δt hours, at approximately t hours past noon, approximately how far does the car travel?

   (b) Write down a Riemann sum representing the total distance the car travels between noon and 2 PM.

   (Don't forget to explain what you are doing. In particular, if your formula includes things like "Δt" or "x_i", you should say what they mean.)

5. (a) A bucket of sand with total mass 2 kg is lifted by a lightweight rope at a constant speed of 10 km/h (kilometers per hour) to the height of 16 m. What is the total work done, in joules (J)?

   (Near the earth's surface, the force of gravity acting on an object of mass m kg is gm newtons, where $g \approx 9.8$.)

   (b) If the sand is leaking out, so that when the bucket reaches height h meters, its mass is $(2 - .01h)$ kilograms, what is the total work done?

6. A solid is generated by revolving around the y-axis the region bounded by $x = 1$, $y = e^x$, the y-axis and the x-axis. Compute its volume.

7. Compute $\int x^3 \cos(x)dx$.