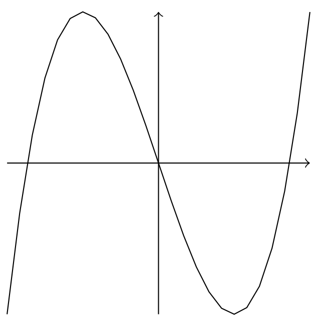
Math 8
Fall 2019
Practice Problems for Exam II

On a real exam, parts of problem (1), and problems (4), (5), (6) and (10) could be short answer problems.

1. The curve in the picture is the graph of the function $y = x^3 - 3x$ in the xy-plane. The picture includes the region of the plane $-2 \le x \le 2$, $-2 \le y \le 2$.



   (a) Give a function $\vec{r}(t)$ parametrizing this curve.

   (b) For your function $\vec{r}$, find $\displaystyle\int_0^1 \vec{r}\,'(t)\,dt$.

   (c) *Without calculating the unit tangent vector at all points*, find all points on the curve at which the unit tangent vector $\vec{T}$ is parallel to the x-axis. For your parametrization, what is the value of $\vec{T}$ at those points?

   (d) *Without calculating the unit normal vector at all points*, find all points on the curve at which the unit normal vector $\vec{N}$ is equal to $\langle 0, 1\rangle$.
   Explain your reasoning.

   (e) Assuming your parametrization is the position function of a moving particle, find the tangential and normal components of the acceleration when the particle is at the point $(1, -2)$.
   These are scalars $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$ for which we write acceleration as $\vec{a} = a_{\mathbf{T}}\vec{T} + a_{\mathbf{N}}\vec{N}$.

   (f) When the particle is at the point $(1, -2)$, is its speed increasing, decreasing, or neither?

   (g) Write down an integral that gives the arc length of the portion of this curve where $-1 \le x \le 1$. Do not evaluate this integral.

2. The curve γ is the intersection of the surfaces $x^2-2x+4y^2+16y=-13$ and $x+2y-z=2$.

   (a) Find a function parametrizing γ.

   (b) Find the curvature of γ at the point $(1,-1,-3)$.

3. TRUE or FALSE? Explain briefly.

   Note: If a statement is true, your explanation could be that it is a definition or a theorem, something that was explicitly stated in the textbook. Otherwise, you should give a brief explanation of why we know it is true.

   If a statement is false, your explanation could be an explanation of why it is not true, an example of a specific case when it is false, or a correction to make it a true statement.

   (a) Suppose a curve γ is defined as the intersection of two given surfaces. If the curvature κ for γ is computed using two different parametrizations, the answer will always be the same.

   (b) If $\vec{r}(t)$ parametrizes any curve on the sphere $x^2+y^2+z^2=1$, then the derivative $\vec{r}'(t)$ is always normal to the position vector $\vec{r}(t)$.

   (c) The curve parametrized by $\vec{r}(t)=\langle t^3,t^3,t\rangle$ is not a smooth curve.

   (d) If a particle travels around a circle, then the normal component of its acceleration is constant.

   (e) Two planes parallel to the same line are parallel.

   (f) Two planes perpendicular to the same plane are parallel.

   (g) If $r(\vec{t})$ is the position function for a particle moving in the plane $x+y+z=3$, then the velocity vector $v(\vec{t})$ is always normal to the vector $\langle 1,1,1\rangle$.

   (h) If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are nonzero vectors such that $\vec{u}\cdot\vec{v}=0$ and $\vec{u}\times\vec{w}=\vec{0}$, then $\vec{v}\cdot\vec{w}=0$.

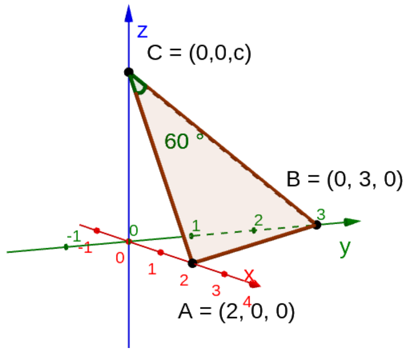
4. Find the vector $(\vec{i}+\vec{j})\times(\vec{i}-\vec{j})$.

5. Find the center and radius of the sphere with equation $x^2+y^2+z^2-2x+4z=164$.

6. Find the equation of the line perpendicular to the plane $x+3y+z=5$ and passing through $(1,0,6)$.

7. Find the distance between the planes with equations $2x-3y+z=4$ and $4x-6y+2z=3$. If the distance is 0, find the angle between the planes.

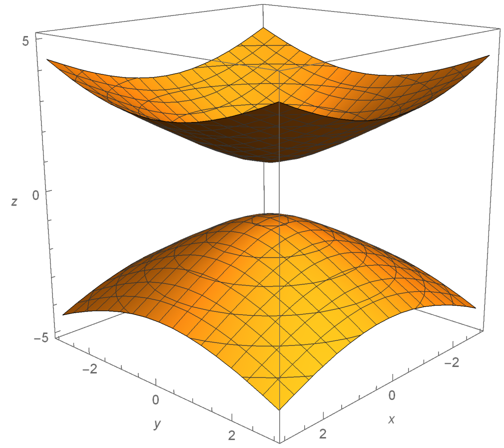
8. What value of c makes this figure possible?



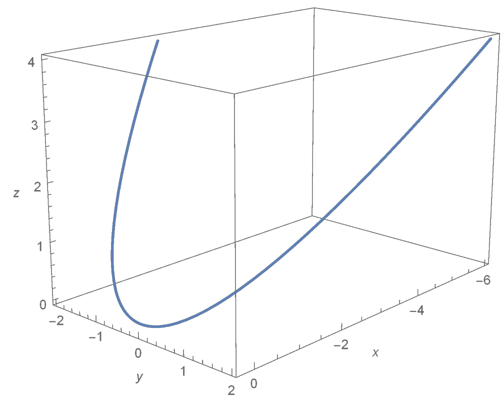
9. An object is traveling counterclockwise along the circle $x^2 + y^2 = 1$, and its speed at time t is $t^2 + 1$. At time $t = 1$ the object is located at the point $(0, 1)$. Find its acceleration at time $t = 1$.

   Hint: First find the tangential and normal components of the acceleration.

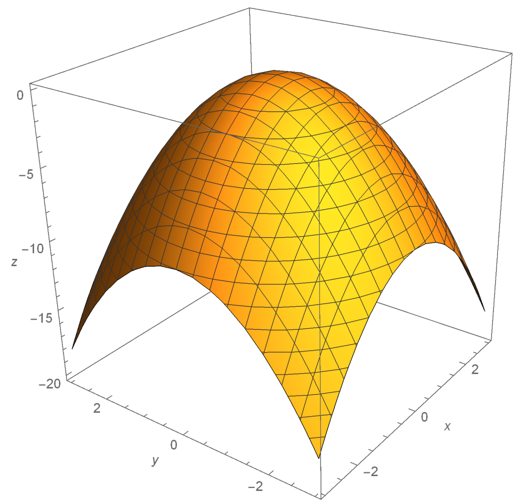
10. By the number of each picture, write the letter of the matching description, equation, or parametrization. The descriptions, the last two pictures, and the space for your answers, are on the next page.



1)



2)



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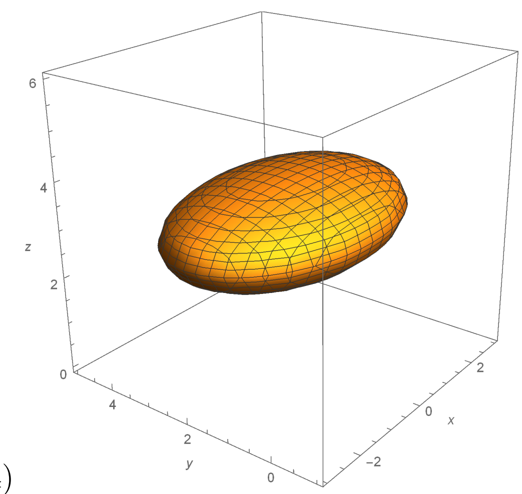

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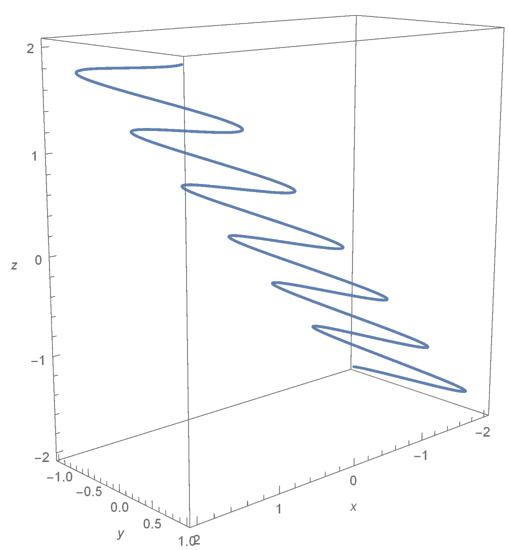

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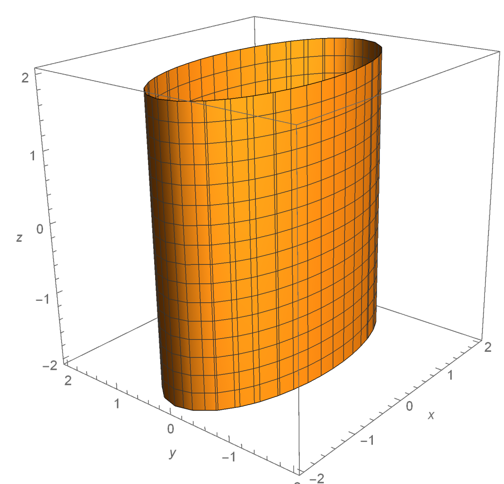
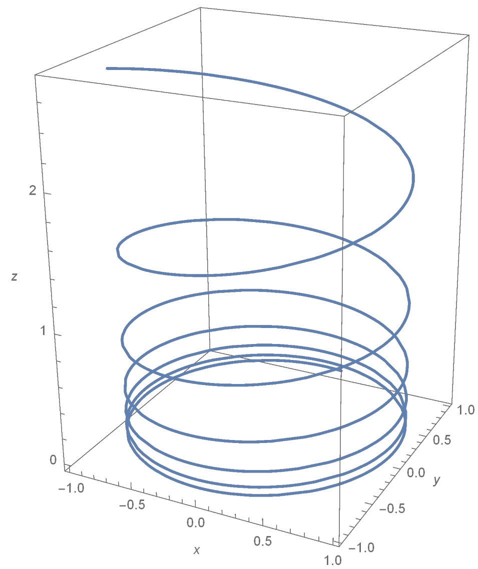
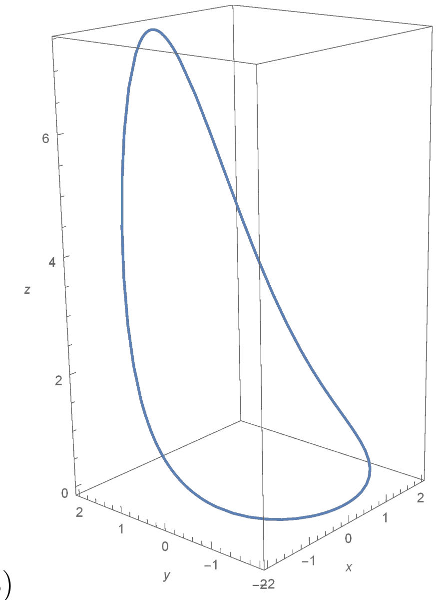


6)



7)



8)

A $x^2 + y^2 + z = 0$

B $z^2 = 1 + x^2 + y^2$

C $z^2 + x^2 = 2 - y^2$

D $\frac{x^2}{9} + \frac{(y-2)^2}{4} + (z-3)^2 = 1$

E $4x - \frac{y}{3} + z = 20$

F $\frac{x^2}{4} + y^2 = 1$

G The intersection of $x = z$ and $y = \sin(x)$

H The intersection of $x^2 + y^2 = 2$ and $z = e^y$

I The intersection of $y^2 = z$ and the plane containing the origin and normal to the vector $\vec{v} = \langle 1, 1, 1 \rangle$

J The intersection of $x = z$ and $z = y^2$

K $\vec{r}(t) = \langle \cos(t), \sin(t), e^t \rangle$

L $\vec{r}(t) = \langle \cos(t), \sin(3t), t \rangle$

WRITE YOUR ANSWERS BELOW

Write by the number of each picture the letter of the matching description, equation, or parametrization:

1) __________ 2)__________ 3) __________ 4)__________

5) __________ 6)__________ 7) __________ 8)__________