Math 8
Fall 2019

Written Homework
Assigned Wednesday, October 9

Note: Standard (not preliminary) written homework is graded on your work and your explanations, not just on your answer.

Explanations are important for many reasons. Being able to communicate what you know shows a depth of understanding beyond that of being able to get the right answer to a problem. Doing the mental work of putting explanations into words helps create that depth of understanding. On exams, we will grade your work and not just your answers, so this is good practice for taking exams.

For all these reasons, be sure to: show all your work; explain your reasoning; use clear English; write neatly so all this effort does not go to waste.

Written homework is always due at 10:00 AM on the following Monday.

In these questions, we use vectors to model the behavior of an object moving along a straight line (in three-dimensional space) with constant speed. Our distance units are meters and our time units are seconds.

If a moving object goes from point P to point Q, its *displacement* vector $\vec{d}$ is the vector from P to Q. That is, you can draw $\vec{d}$ with its tail at P and its head at Q. You can find the entries of $\vec{d}$ by subtracting the corresponding coordinates of P from those of Q.

> **Example:** An object starts from point $(3, 2, 4)$ and moves to point $(0, 2, 0)$. Its displacement vector is
>
> $$\vec{d} = \langle 0 - 3,\ 2 - 2,\ 0 - 4 \rangle = \langle -3,\ 0,\ -4 \rangle$$

If this moving object goes from point P to point Q in a straight line at a constant speed, its *velocity* vector $\vec{v}$ is the vector whose direction is the direction of motion (that is, $\vec{v}$ has the same direction as $\vec{d}$) and whose length is the speed at which the object moves (the distance between P and Q divided by the time it took the object to get from P to Q).

> **Example:** Suppose the object in the previous example moved in a straight line at constant speed and took 15 seconds to move from P to Q. A unit vector in the direction of its motion is
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> $$\vec{u} = \frac{1}{|\vec{d}|}\vec{d} = \frac{1}{\sqrt{3^2 + 0^2 + 4^2}} \langle -3,\ 0,\ -4 \rangle = \left\langle -\frac{3}{5},\ 0,\ -\frac{4}{5} \right\rangle$$
>
> The distance it travels is the distance from P to Q, or the length of $\vec{d}$, which is 5, so its speed is
>
> $$s = \frac{5}{15} = \frac{1}{3}$$

Its velocity vector has the direction of $\vec{u}$ (the direction of motion) and length s (the speed), so its velocity is

$$\vec{v} = s\vec{u} = \frac{1}{3}\left\langle -\frac{3}{5},\, 0,\, -\frac{4}{5}\right\rangle = \left\langle -\frac{3}{15},\, 0,\, -\frac{4}{15}\right\rangle$$

An object moving in a straight line with constant speed is said to be moving at constant velocity. (Since this is calculus, we will soon learn how to model the behavior of objects moving with continuously changing velocity.)

Problems: (1, 2, and 4 are short answer problems; no explanation is needed. Be sure to explain your answers to 3 and 5.)

1. Object A moves with constant velocity from point $(1, 2, 1)$ to point $(4, -10, -3)$ in 65 seconds.
   (a) Find the distance it travels.
   (b) Find its displacement vector.
   (c) Find a unit vector in its direction of motion.
   (d) Find its speed.
   (e) Find its velocity vector.
2. Object B starts at point $(0, 3, 2)$ and moves for 3 seconds with the same constant velocity as object A.
   (a) What is its speed?
   (b) Find the distance it travels.
   (c) Give a unit vector in its direction of motion.
   (d) Find its displacement vector.
   (e) Find its final position.
3. Object C starts at point (a, b, c) and moves for t seconds with the same constant velocity as objects A and B. Find its final position.
4. A vector $\vec{v}$ can always be written as $\vec{v} = |\vec{v}|\vec{u}$, where $|\vec{v}|$ is the length of $\vec{v}$, and $\vec{u}$ is a unit vector in the same direction as $\vec{v}$.
   (a) If $\vec{v}$ is a displacement vector, what are the physical interpretations of $|\vec{v}|$ and $\vec{u}$?
   (b) If $\vec{v}$ is a velocity vector, what are the physical interpretations of $|\vec{v}|$ and $\vec{u}$?
5. Object D starts at point (a, b, c) and moves with constant velocity $\vec{v} = \langle v_x,\, v_y,\, v_z\rangle$ for t seconds. Find its final position.