

Preliminary Homework  
Assigned Wednesday, November 6

Note: Preliminary homework is always graded credit or no credit. **You get full credit for completing the assignment, whether or not your answers are correct.** The purpose of preliminary homework is to start you thinking about the topic of the next class.

You may use your preliminary homework in activities with your classmates. You should be sure to think about these questions so you will be prepared.

Preliminary homework is always due at the *beginning* of class.

**Assignment:** A surface  $S$  has the equation  $z = f(x, y)$ . At  $(x, y) = (1, 2)$  we have

$$z = 45 \quad \frac{\partial z}{\partial x} = 2 \quad \frac{\partial z}{\partial y} = 1$$

A bug is crawling on the surface  $S$ , and a light shining directly down through  $S$  (which is transparent) casts the bug's shadow on the  $xy$ -plane; the position of the shadow is  $\vec{r}(t)$ . At time  $t_0$ , the bug's shadow has position  $\vec{r}(t_0) = \langle 1, 2 \rangle$  and velocity  $r'(t_0) = \langle 3, -1 \rangle$ .

1. Find an equation for the tangent plane to  $S$  at the point  $(1, 2, 45)$ .
2. Use the tangent approximation

$$\vec{r}(t) \approx \vec{r}(t_0) + (t - t_0)\vec{r}'(t)$$

to approximate the shadow's position at time  $t + \Delta t$ , where  $\Delta t$  is a very small change in  $t$ .

3. Use the equation of the tangent plane to  $S$  to approximate the bug's new  $z$ -coordinate.
4. Show that your answer is

$$45 + \left( \left\langle \frac{\partial z}{\partial x}(\vec{r}(t_0)), \frac{\partial z}{\partial y}(\vec{r}(t_0)) \right\rangle \cdot \vec{r}'(t_0) \right) \Delta t.$$