Math 8
Fall 2019

Written Homework
Assigned Friday, November 8

Note: Standard (not preliminary) written homework is graded on your work and your explanations, not just on your answer.

Explanations are important for many reasons. Being able to communicate what you know shows a depth of understanding beyond that of being able to get the right answer to a problem. Doing the mental work of putting explanations into words helps create that depth of understanding. On exams, we will grade your work and not just your answers, so this is good practice for taking exams.

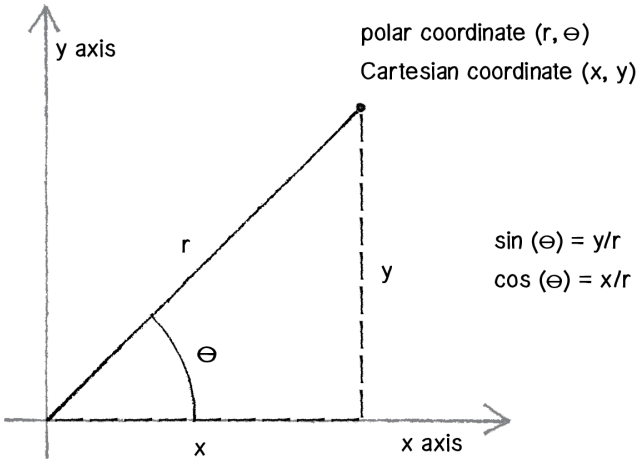
For all these reasons, be sure to: show all your work; explain your reasoning; use clear English; write neatly so all this effort does not go to waste.

Written homework is always due at 10:00 AM on the following Monday.

Assignment: For any point (x, y) in the xy-plane with $x > 0$, we can define the functions $r(x, y)$, the distance between (x, y) and the origin, and $\theta(x, y)$, the angle from the positive x-axis to the ray from $(0, 0)$ to (x, y), by

$$r(x, y) = \sqrt{x^2 + y^2} \qquad \theta(x, y) = \tan^{-1}\left(\frac{y}{x}\right).$$

$r(x, y)$ and $\theta(x, y)$ are called polar coordinates, and are shown in the figure below:



1. At time $t = 0$, a moving point $P = (x, y)$ is located at $(2, 2)$ and moving with velocity vector $\langle \frac{\partial x}{\partial t}, \frac{\partial y}{\partial t} \rangle = \langle 2, -1 \rangle$ (so both x and y are changing with time).

   Compute the following derivatives at $t = 0$:

   (a) $\frac{\partial}{\partial t} r$.

   (b) $\frac{\partial}{\partial t} \theta$.

2. Suppose instead that at $t = 0$, the particle is located at $(2, 2)$ but now has velocity $\langle 2, 2\rangle$. Show that at $t = 0$, $\frac{\partial}{\partial t}\theta = 0$.

3. Now suppose that when $t = 0$, the particle at $(2, 2)$ has velocity $\langle -1, 1\rangle$. Show that $\frac{\partial}{\partial t}r = 0$ at $t = 0$.

4. Provide an intuitive explanation for the results of 2 and 3. It may be useful to draw a picture.