Math 8
Fall 2019

Preliminary Homework
Assigned Wednesday, September 18

Note: Preliminary homework is always graded credit or no credit. **You get full credit for completing the assignment, whether or not your answers are correct, as long as your work shows you have thought about the problem.** The purpose of preliminary homework is to start you thinking about the topic of the next class.

You may use your preliminary homework for in-class activities with your classmates. You should be sure to think about these questions so you will be prepared.

Preliminary homework is always due at the *beginning* of the next class.

The degree 1 Taylor polynomial approximation to $f(x) = \sin x$ centered at 0 is

$$P_1(x) = x.$$

Therefore we may say that

$$\sin .01 \approx .01.$$

(Note that .01 means .01 radians.) But how close is this approximation?

First we ask how large $f(.01)$ could possibly be. We know that $f(0) = 0$ and $f'(0) = 1$. We also know that $-1 \leq f''(x) \leq 1$ for every x.

The largest possible value of $f(.01)$ for a function with these properties will occur when f grows as fast as possible between 0 and .01. That will happen when $f'(x)$ is as large as possible. The largest possible values of $f'(x)$ will occur when $f''(x)$ is as large as possible. The largest $f''(x)$ can possibly be is 1. So by assuming $f''(x) = 1$, we can find an upper bound on how large $f(.01)$ could be.

Homework:

1. Find a degree 2 polynomial $P(x)$ with the same value and derivative as $f(x) = \sin x$ at 0 and constant second derivative $P''(x) = 1$.

   How large could $\sin .01$ possibly be?

2. Find a degree 2 polynomial $P(x)$ with the same value and derivative as $f(x) = \sin x$ at 0 and constant second derivative $P''(x) = -1$.

   How small could $\sin .01$ possibly be?

3. How large could the difference $|\sin .01 - .01|$ possibly be?