Math 8

Fall 2019

Units of Curvature

If you don't care about the units of curvature, if you are happy with "curvature κ means bends as much as a circle of radius $\frac{1}{\kappa}$", you can stop reading here.

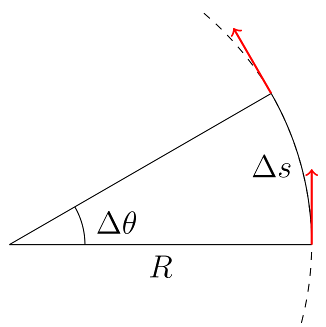
If you just want to know the official answer, but knowing that answer will have nothing to do with your understanding of curvature, the official answer is that the units of curvature are inverse meters, $\frac{1}{m}$. In imperial units, this would be inverse feet, $\frac{1}{ft}$. You can look at the web page

`http://jlmartin.faculty.ku.edu/~jlmartin/courses/math223-F10/units.pdf`

for a discussion of this. The author of that web page is Jeremy L. Martin, Professor of Mathematics at the University of Kansas.

However, if you want to think of curvature in units that make sense, you can usefully think of the units of curvature as radians per meter. (Strictly speaking, radians are "dimensionless units" or "pure numbers." This is why we get inverse meters.)

To see why, consider traveling around a circle of radius R for some short distance Δs, with a corresponding angle $\Delta\theta$, as in the picture.



The vectors drawn in red are the unit tangent vectors at the beginning and end of the path. The angle between them is the same as the angle between the radii they are normal to, or $\Delta\theta$, measured in radians. You can take this as a measure of how much the direction of motion has changed.

The length of the path, or the distance traveled, is the angle times the radius, which is $\Delta s = R\Delta\theta$, measured in meters. Therefore we can say the rate of change of direction with respect to arc length is

$$\frac{\Delta\theta \text{ (radians)}}{\Delta s \text{ meters}} = \frac{\Delta\theta \text{ (radians)}}{R\Delta\theta \text{ meters}} = \frac{1}{R}\frac{\text{(radian)}}{\text{meters}}.$$

This is the curvature κ of a circle of radius R.