Math 9, Fall 2001, Practice problems for final exam

1. Which of the following is the general solution of
$$ty' - y = t^2 e^{-t} \quad , \quad t > 0\ ?$$
(a) $y(t) = t(e^{-t} + c)$ (b) $y(t) = t(-e^{-t} + c)$ (c) $y(t) = t(e^{t} + c)$ (d) $y(t) = t(-e^{t} + c)$

2. The **explicit** solution of the initial value problem
$$\frac{dy}{dx} = \frac{e^x + e^{-x}}{2y+2} \quad , \quad y(0) = -2$$
is
(a) $y(x) = -1 + \sqrt{e^x - e^{-x} + 1}$
(b) $y(x) = -2 + \sqrt{e^x - e^{-x}}$
(c) $y^2 + 2y = e^x - e^{-x}$
(d) $y = \ln|2y+2|\,(e^x - e^{-x})$
(e) $y(x) = -1 - \sqrt{e^x - e^{-x} + 1}$

3. A college graduate decides to pay her loans of $\$100,000$ in 5 years. The interest rate applied to her debt is 10% **annually** and in order to see if this plan is reasonable she must compute the necessary **monthly** payment x. Let $S(t)$ be her debt at time t. Assuming that interest is compounded continuously, which of the following initial value problems models the situation and can be used to compute x?
(a) $\frac{dS}{dt} = 0.1\,S + 12\,x \quad , \quad S(5) = 0$
(b) $\frac{dS}{dt} = 0.1\,S - x \quad , \quad S(5) = 0$
(c) $\frac{dS}{dt} = -0.1\,S + 12\,x \quad , \quad S(0) = 100,000$
(d) $\frac{dS}{dt} = 0.1(S - 12\,x) \quad , \quad S(0) = 100,000$
(e) $\frac{dS}{dt} = 0.1\,S - 12\,x \quad , \quad S(0) = 100,000$

4. For the following series determine whether they are convergent or divergent.

   (a)
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n+1}$$

   (b)
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3+n}{5+n}$$

   (c)
$$\sum_{n=1}^{\infty} \frac{n+1}{n!}$$

5. Find all the values of p for which the series $\sum_{n=1}^{\infty} e^{pn}$ converges.

6. **Compute** (that is, give full justification) the MacLaurin series of $\arctan(x)$.

7. The angle between the vectors $\langle 6,\ 1,\ \sqrt{3}\rangle$ and $\langle 1,\ 1,\ \sqrt{3}\rangle$ is

   (a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

8. The only one of the following which is parallel to the line

$$\mathbf{r}(t) = \langle t-1,\ 2t,\ 3t+1\rangle$$

   is

   (a) $\mathbf{r}(t) = \langle \cos t,\ 2\sin t,\ 3t\rangle$

   (b) $\mathbf{r}(t) = \langle \frac{t}{2},\ \frac{t}{4},\ \frac{t}{6}\rangle$

   (c) $\frac{x}{3} = \frac{y}{2} = z$

   (d) $\mathbf{r}(t) = \langle -2t,\ -4t+1,\ -6t\rangle$

9. Consider the points A(1,0,1), B(1,2,4), and C(2,1,1).

   (a) Find the equation of the plane containing these three points.

   (b) Find the area of the triangle with these three points as vertices.

10. Find parametric equations for the tangent line to the curve

$$\mathbf{r}(t) = e^{t-1}\,\mathbf{i} + t^2\,\mathbf{j} + (1-t)\,\mathbf{k}$$

    at the point $(1, 1, 0)$.

11. Find the length of the curve $\mathbf{r}(t) = 6t\ \mathbf{i} + 3\sqrt{2}\,t^2\ \mathbf{j} + 2t^3\ \mathbf{k}$, for $0 \le t \le 1$.

12. Find the point P where the helix $\mathbf{r}(t) = \langle \cos t, \sin t, \frac{1}{2\pi}t\rangle$ intersects the hemisphere $z = \sqrt{2 - x^2 - y^2}$. Compute the tangent line to the helix at this point of intersection.

13. Find the limit or show that it does not exist:

$$\lim_{(x,y)\to(0,1)} \frac{xy}{x+y-1}\ .$$

14. Suppose

$$f(x, y, z) = x\sin(\pi yz).$$

    (a) Find the direction in which f is increasing most rapidly at the point $(2, 1, 2)$.

    (b) Find the value of the maximum rate of increase of f at the point $(2, 1, 2)$.

15. Find and classify all local extrema of the function

$$f(x, y) = x^3 + 3xy - y^3.$$

16. Use the method of Lagrange multipliers to find the maximum and minimum values of

$$f(x, y) = x^2 + y$$

    on the circle $x^2 + y^2 = 1$.

17. Find the absolute minimum and maximum of the function $f(x, y) = x^3 + y^2$ on the disk $x^2 + y^2 \le 1$. Specify all points at which the absolute minimum and maximum occur.