

Math 9 Fall 2001 Final Exam

Sunday December 9, 2001

1. [20 points] Determine whether the following limit exists and, if it exists, find its value:

$$\lim_{(x,y) \rightarrow (1,1)} \frac{y \sin(x-1)}{x+y-2}.$$

2. [15 points] Superman flies through space with coordinate functions $x(t) = 2t^2$, $y(t) = -t^2$, $z(t) = -2t^2$. What is the total distance he travels from time $t = 0$ to $t = 2$?
3. [15 points] Find the general solution to the second-order differential equation

$$y'' - 5y' + 6y = 0.$$

4. [20 points] Let $f(x, y) = x^2 + xy + y^2$.

- (a) Find the maximum and minimum values of f along the circle $x^2 + y^2 = 1$.
- (b) Find the extreme points and values for f in the disk D consisting of the points (x, y) such that $x^2 + y^2 \leq 1$.

5. [20 points] Find the Taylor series of $e^{x/2}$ expanded around $a = 2$ and find its radius of convergence.

6. [20 points] Let $f(x, y, z) = x^2 + y^2 + z^2 - 2x - 2y - 2z + 3$, and let S be a level surface of f given by $f(x, y, z) = 4$. Let P be a point $(1, 1, 3)$ on S .

- (a) Find the equation of the plane that is tangent to S at the point P .
- (b) Find the parametric equations of the line that is normal to S at P .

Justify your answers.

7. [20 points] Find all the local minima, maxima, and saddle points of

$$f(x, y) = x^3 + y^3 - x - 3y + 3.$$

Find also the values of f at these points. Justify your answers.

8. [20 points] Let $f(x, y)$ be a differentiable function of two variables. Given a unit vector \vec{u} , let $D_{\vec{u}}f$ be the directional derivative of f in the direction \vec{u} .

- (a) For which direction \vec{u} is $D_{\vec{u}}f$ maximal?
- (b) What is the maximum value?

To get any credit for this problem, you must prove your answers.