CROSS PRODUCT WORKSHEET

APRIL 22, 2019

Theorem (Properties of the Cross Product)**.** Let $\vec{u}, \vec{v}$, and $\vec{w}$ be vectors and c be a scalar.

(i) $\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$ (anticommutativity)
(ii) $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$ (distributivity)
(iii) $c(\vec{u} \times v) = (c\vec{u}) \times \vec{v} = \vec{u} \times (c\vec{v})$ (scaling property)
(iv) $\vec{u} \times \vec{0} = \vec{0}$
(v) $\vec{v} \times \vec{v} = \vec{0}$
(vi) $\vec{u} \cdot (\vec{v} \times \vec{w}) = (\vec{u} \times \vec{v}) \cdot \vec{w}$ (triple scalar product)

1. Given vectors $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$, show that $\vec{u} \times \vec{v}$ is orthogonal to $\vec{u}$.

2. Using only the geometric interpretation of the cross product and the right-hand rule (i.e., no determinants), compute the following cross products.

$\hat{\imath} \times \hat{\jmath} =$ ________ $\hat{\jmath} \times \hat{\imath} =$ ________

$\hat{\jmath} \times \hat{k} =$ ________ $\hat{k} \times \hat{\jmath} =$ ________

$\hat{k} \times \hat{\imath} =$ ________ $\hat{\imath} \times \hat{k} =$ ________

3. Let $\vec{u} = \langle 3, 2, -1 \rangle$ and $\vec{v} = \langle 1, 1, 0 \rangle$.
   (a) Compute $\vec{u} \times \vec{v}$.

   (b) Compute $\vec{v} \times \vec{u}$.

   (c) Sketch $\vec{u}$, $\vec{v}$, $\vec{u} \times \vec{v}$ and $\vec{v} \times \vec{u}$.

4. Consider the points $A = (3, -1, 2)$, $B = (2, 1, 5)$, and $C = (1, -2, -2)$.
   (a) Find the area of the parallelogram $ABCD$ with adjacent sides $\vec{AB}$ and $\vec{AC}$.

   (b) Find the area of the triangle ABC.

   (c) Find the distance from the point A to the line BC.