Math 8, Winter 2005

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Series

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where $\{a_n\}$ is an infinite sequence.

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- A series *coverges* if the sequence of partial sums $\{s_m\}$ converges. Otherwise the series *diverges*.

The test for divergence

One easy consequence of this definition is the test for divergence:

Consider a series $\sum_{i=k}^{\infty} a_n$ and the sequence of summands, $\{a_n\}$. If a_n converges to $L \neq 0$ then $\sum_{i=k}^{\infty} a_n$ diverges.

The test for divergence

It is NOT TRUE that

> If $a_n \to 0$ then $\sum_{i=k}^{\infty} a_n$ converges.

The harmonic series is a counterexample:

$$\frac{1}{n} \to 0$$

but

$$\sum_{i=1}^{\infty} \frac{1}{n} \quad \text{diverges}$$

Examples

Examples

- We've seen two examples via improper integrals:

$$\sum_{i=1}^{\infty} \frac{1}{n} \quad \text{diverges}$$

and

$$\sum_{i=1}^{\infty} \frac{1}{n^2} \quad \text{converges}$$

More examples

- Does

$$\sum_{i=2}^{\infty} \frac{1}{n^2 - n}$$

converge or diverge?

- Let a be a real number. For which values of r does

$$\sum_{i=0}^{\infty} ar^n$$

converge?

Geometric Series

Suppose a is a real number and $|r| < 1$ then

$$\sum_{i=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$$

Geometric series examples

- $$\sum_{i=1}^{\infty} 10\frac{3^n}{4^{n-1}}$$
- $$\sum_{i=0}^{\infty} -2\frac{1}{4^n}$$
- $$\sum_{i=2}^{\infty} \frac{9^{n-1}}{10^{n-1}}$$

Integral Test

We've already used integrals to help determine the convergence of some series. We can now formalize this into a test:

Suppose f is a continuous, positive, decreasing function defined for $1 \leq x < \infty$ and let $a_n = f(n)$. Then

- If $\int_1^\infty f(x)\,dx$ converges, then $\sum_{i=1}^\infty a_n$ converges as well.
- If $\int_1^\infty f(x)\,dx$ diverges, then $\sum_{i=1}^\infty a_n$ diverges as well.

Examples

- Suppose $p > 0$, for which values of p does

$$\sum_{i=1}^{\infty} \frac{1}{n^p}$$

converge?

-

$$\sum_{i=1}^{\infty} \frac{\ln(n)}{n}$$

-

$$\sum_{i=1}^{\infty} ne^{-n}$$