Worksheet #1

Use integration by parts to perform the indicated integrations.

(1) $\int t(2t+7)^{1/3}dt$

Solution: Let $\begin{array}{cc} u = t & v = \frac{3}{8}(2t+7)^{4/3} \\ du = dt & dv = (2t+7)^{1/3}\,dt \end{array}$

Then

$$\begin{aligned}\int t(2t+7)^{1/3}dt &= \frac{3}{8}t(2t+7)^{4/3} - \int \frac{3}{8}(2t+7)^{4/3}\,dt \\ &= \frac{3}{8}t(2t+7)^{4/3} - \left(\frac{3}{8}\right)\left(\frac{3}{14}\right)(2t+7)^{7/3} + C\end{aligned}$$

(2) $\int \frac{\ln x}{x^2}dx$

Solution: Let $\begin{array}{cc} u = \ln x & v = -x^{-1} \\ du = \frac{1}{x}dx & dv = x^{-2}. \end{array}$ Then

$$\begin{aligned}\int \frac{\ln x}{x^2}dx &= -x^{-1}\ln x + \int x^{-2}dx \\ &= -x^{-1}\ln x - x^{-1} + C\end{aligned}$$

(3) $\int_0^{\pi} x^2 \cos x dx$

Solution: Let $\begin{array}{cc} u = x^2 & v = \sin x \\ du = 2xdx & dv = \cos x dx. \end{array}$ Then

$$\int_0^{\pi} x^2\cos x dx = x^2 \sin x|_0^{\pi} - 2\int_0^{\pi} x\sin x dx.$$

We must integrate by parts again. Letting $\begin{array}{cc} u = x & v = -\cos x \\ du = dx & dv = \sin x dx, \end{array}$ and integrating, we get

$$\begin{aligned}\int_0^{\pi} x^2\cos x dx &= x^2\sin x|_0^{\pi} - 2\left(x\cos x|_0^{\pi} + \int_0^{\pi}\cos x dx\right) \\ &= x^2\sin x + 2x\cos x - 2\sin x|_0^{\pi} \\ &= \left(\pi^2(0) - 0\right) + 2\left(\pi(1) - 0\right) - 2\left(0 - 0\right) \\ &= 2\pi.\end{aligned}$$

(4) $\int e^{\alpha z} \cos(\beta z) dz$

Solution: Let $\begin{array}{cc} u = \cos(\beta z) & v = \alpha^{-1} e^{\alpha z} \\ du = -\beta \sin(\beta z) dz & dv = e^{\alpha z} dz. \end{array}$ Then

$$\int e^{\alpha z} \cos(\beta z) dz = \alpha^{-1} e^{\alpha z} \cos(\beta z) + \beta \alpha^{-1} \int e^{\alpha z} \sin(\beta z) dz.$$

We must integrate by parts again. Letting $\begin{array}{cc} u = \sin(\beta z) & v = \alpha^{-1} e^{\alpha z} \\ du = \beta \cos(\beta z) dz & dv = e^{\alpha z} dz, \end{array}$ we get

$$\int e^{\alpha z} \cos(\beta z) dz = \alpha^{-1} e^{\alpha z} \cos(\beta z) + \beta \alpha^{-1} \left(\alpha^{-1} e^{\alpha z} \sin(\beta z) - \alpha^{-1} \beta \int e^{\alpha z} \cos(\beta z) dz \right).$$

Collecting like terms on the left-hand side and dividing by the constant, we see that

$$\left(1 + \beta^2 \alpha^{-2}\right) \int e^{\alpha z} \cos(\beta z) dz = \left(\alpha^{-1} e^{\alpha z} \cos(\beta z) + \beta \alpha^{-2} e^{\alpha z} \sin(\beta z)\right) + C.$$

Solving for the integral on the left hand side, we get our answer:

$$\int e^{\alpha z} \cos(\beta z) dz = \left(\alpha^{-1} e^{\alpha z} \cos(\beta z) + \beta \alpha^{-2} e^{\alpha z} \sin(\beta z)\right) \left(1 + \beta^2 \alpha^{-2}\right)^{-1}.$$