Worksheet #23

(1) Find the equation of the tangent plane to the surface $z = 2e^{3y}\cos(2x)$ at $(\pi/3, 0, -1)$.
Solution: We first need to compute $\mathbf{n} =< -f_x(\pi/3,0), -f_y(\pi/3,0), 1 >$.

$$f_x = -4e^{3y}\sin(2x) \quad f_x(\pi/3,0) = -2\sqrt{3}$$
$$f_y = 6e^{3y}\cos(2x) \quad f_x(\pi/3,0) = -3$$

The plane is given by

$$< 2\sqrt{3}, 3, 1 > \cdot < x - \pi/3, y, z+1 >= 0$$

Expanded, the equation is

$$z = -1 - 2\sqrt{3}(x - \pi/3) - 3y.$$

(2) Find all points on the surface $z = x^2 - 2xy - y^2 - 8x + 4y$, where the tangent plane is horizontal.
Solution: The tanget plane being horizontal implies $\mathbf{n} =< -f_x, -f_y, 1 >=< 0, 0, 1 >$. This means that $f_x = 0$ and $f_y = 0$. Creating these equations,

$$f_x = 2x - 2y - 8 = 0 \quad f_y = -2x - 2y + 4 = 0$$

Adding these two equations, we find $y = 1$. Plugging this into the either other equation, $x = 5$. So there is only one point $(5, 1, -22)$.

(3) Use the total differential dz to approximate the change in z as (x, y) moves from P to Q where $z = \ln(x^2y)$ where $P(-2, 4)$ and $Q(-1.98, 3.96)$.
Solution: $dx = 0.02$ and $dy = -0.04$. Computing the differential, we find

$$dz = z_x dx + z_y dy = \frac{2xy}{x^2y}dx + \frac{x^2}{x^2y}dy = \frac{2}{x}dx + \frac{1}{y}dy = -0.03.$$

(4) In determining the specific gravity of an object, its weight in air is found to be $A = 36$ lbs and its weight in water is $W = 20$ lbs, with a possible error in each measurement of 0.02 lb. Approximate the error in calculating the specific gravity S, where $S = A/(A - W)$.
Solution:

$$\begin{aligned} dS &= S_A dA + S_W dW \\ &= (-\frac{A}{(A-W)^2} + \frac{1}{A-W})dA + \frac{A}{(A-W)^2}dW \\ &= (\frac{-36}{16^2} + \frac{1}{16})0.002 + \frac{36}{16^2}(0.02) \\ &= \frac{0.02}{16} \end{aligned}$$