Worksheet #24

(1) Find $\frac{dw}{dt}$ where $w = e^x \sin y + e^y \sin x$, $x = 3t$ and $y = 2t$.
Solution: Use the chain rule.

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}$$
$$= (e^x \sin y + e^y \cos x)\frac{dx}{dt} + (e^x \cos y + e^y \sin x)\frac{dy}{dt}$$
$$= (e^x \sin y + e^y \cos x)(3) + (e^x \cos y + e^y \sin x)2 \qquad = 3\left(e^{3t}\sin(2t) + e^{2t}\cos(3t)\right) + 2\left(e^{3t}\cos(2t) + e^{2t}\sin(3t)\right)$$

(2) Find $\frac{\partial z}{\partial t}$ where $z = \ln(x+y) - \ln(x-y)$, $x = te^s$ and $y = e^{st}$. Express your answer in terms of s and t.
Solution: Use the chain rule.

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$
$$= \left(\frac{1}{x+y} - \frac{1}{x-y}\right)\frac{\partial x}{\partial t} + \left(\frac{1}{x+y} + \frac{1}{x-y}\right)\frac{\partial y}{\partial t}$$
$$= \left(\frac{1}{te^s + e^{st}} - \frac{1}{te^s - e^{st}}\right)e^s + \left(\frac{1}{te^s + e^{st}} + \frac{1}{te^s - e^{st}}\right)se^{st}$$

(3) If $w = x^2y + z^2$, $x = \rho\cos\theta\sin\phi$, $y = \rho\sin\theta\sin\phi$, and $z = \rho\cos\phi$, find $\frac{\partial w}{\partial \rho}$ evaluated at $\rho = 2$, $\theta = \pi$ and $\phi = \pi/2$.
Solution: Use the chain rule.

$$\frac{\partial w}{\partial \rho} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial \rho} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \rho} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial \rho}$$
$$= (2xy)\frac{\partial x}{\partial \rho} + x^2\frac{\partial y}{\partial \rho} + 2z\frac{\partial z}{\partial \rho}$$
$$= (2\rho^2\cos\theta\sin^2\phi\sin\theta)(\cos\theta\sin\phi) + (\rho\cos\theta\sin\phi)^2\cos\theta\sin\phi + 2\rho\cos\phi(\cos\phi)$$
$$= \left(2(2)^2\cos(\pi)\sin^2(\pi/2)\sin(\pi)\right)(\cos(\pi)\sin(\pi/2)) +$$
$$(2\cos(\pi)\sin(\pi/2))^2\cos(\pi)\sin(\pi/2) + 2(2)\cos(\pi/2)(\cos(\pi/2))$$
$$= 0 + (2(-1)(1))^2(-1)(1) + 2(2)(0)(0)$$
$$= -4$$

(4) If $3x^2z + y^3 - xyz^3 = 0$, find $\frac{\partial z}{\partial x}$.
Solution: Use implicit differentiation.

$$3(x^3\frac{\partial z}{\partial x} + 2xz) - (3xyz^2\frac{\partial z}{\partial x} + yz^3) = 0$$

Solving for $\frac{\partial z}{\partial x}$, we get

$$\frac{\partial z}{\partial x} = \frac{-6xz + yz^3}{3x^3 - 3xyz^2}$$