Worksheet #2

Perform the indicated integration

(1) $\int \sin^4(2x)dx$

Solution:

$$\begin{aligned}
\int \sin^4(2x)dx &= \int \left(\sin^2(2x)\right)^2 dx \\
&= \int \left(1/2\left(1-\cos(4x)\right)\right)^2 dx \quad \text{by half} - \text{angle formula} \\
&= \frac{1}{4}\int \left(1 - 2\cos(4x) + \cos^2(4x)\right) dx \\
&= \frac{1}{4}\int \left(1 - 2\cos(4x) + 1/2\left(1+\cos(8x)\right)\right) dx \quad \text{by half} - \text{angle formula} \\
&= \frac{1}{4}\left(x - \frac{1}{4}\sin(4x) + \frac{1}{2}x + \frac{1}{16}\sin(8x)\right) + C
\end{aligned}$$

(5) $\int \tan y \, dy$

Solution:

$$\begin{aligned}
\int \tan y \, dy &= \int \frac{\sin y}{\cos y} dy \\
&= -\int \frac{1}{u} du = -\ln|u| + C \\
&= \ln\left|\frac{1}{\cos(y)}\right| + C = \ln|\sec y| + C
\end{aligned}$$

where $u = \cos y$, and $du = \sin y dy$.

(2) $\int \tan^3(2x)dx$

Solution:

$$
\begin{aligned}
\int \tan^3(2x)dx &= \int \tan^2(2x)\tan(2x)dx \\
&= \int \left(\sec^2(2x) - 1\right)\tan(2x)dx \\
&= \int \left(\sec^2(2x)\tan(2x) - \tan(2x)\right)dx \\
&= \int \frac{u}{2}du - \int \tan(2x)dx \\
&= \frac{u^2}{4} - \frac{1}{2}\ln|\sec(2x)| + C \\
&= \frac{\sec^2(2x)}{4} - \frac{1}{2}\ln|\sec(2x)| + C,
\end{aligned}
$$

where $u = \sec(2x)$. (Thus $du = 2\sec(2x)\tan(2x)dx$.

(3) $\int \cos^6\theta \sin^2\theta d\theta$

Solution:

$$\begin{aligned}
\int \cos^6\theta \sin^2\theta d\theta &= \int (\cos^2\theta)^3 \sin^2\theta d\theta \\
&= \int \left(\frac{1}{2}\left(1+\cos(2\theta)\right)\right)^3 \left(\frac{1}{2}\left(1-\cos(2\theta)\right)\right) d\theta \\
&= \frac{1}{16}\int \left(1+2\cos(2\theta)-2\cos^3(2\theta)-\cos^4(2\theta)\right) d\theta \\
&= \frac{1}{16}\int \left(1+2\sin^2(2\theta)\cos(2\theta)-(\cos^2(2\theta))^2\right) d\theta \\
&= \frac{1}{16}\int \left(1+2\sin^2(2\theta)\cos(2\theta)-\left(\frac{1}{2}(1+\cos(4\theta)\right)^2\right) d\theta \\
&= \frac{1}{16}\int \left(1+2\sin^2(2\theta)\cos(2\theta)-\frac{1}{4}\left((1+2\cos(4\theta)+\cos^2(4\theta)\right)\right) d\theta \\
&= \frac{1}{16}\int \left(1+2\sin^2(2\theta)\cos(2\theta)-\frac{1}{4}\left((1+2\cos(4\theta)+\frac{1}{2}(1+\cos(8\theta))\right)\right) d\theta \\
&= \frac{1}{16}\left(x+1/3\sin^3(2\theta)-\frac{1}{4}\left((x+\frac{1}{2}\sin(4\theta)+\frac{1}{2}(x+\frac{1}{8}\sin(8\theta))\right)\right)+C
\end{aligned}$$

(4) $\int \frac{\sec^2 q}{\tan^4 q} dq$

Solution: We proceed by u-subsitution. Let $u = \tan q$, then $du = \sec^2 q dq$.

$$\begin{aligned}
\int \tan^{-4} q \sec^2 q dq &= \int u^{-4} du \\
&= -\frac{1}{3}u^{-3} + C \\
&= -\frac{1}{3}\frac{1}{\tan^3 q} + C \\
&= -\frac{1}{3}\cot^3 q + C
\end{aligned}$$