Worksheet #8

(1) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}}$
Also determine which partial sum is accurate to 0.01.

Solution: Note that the $\lim_{n\to\infty}\frac{1}{\sqrt{n}} = 0$ and $\frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n+1}} > 0$ for all n. Thus by the alternating series test, the series converges.

We need to find N such that $\sqrt{N+1} > 100$. Thus $N > 100^2 - 1$.

(2) Does $\sum_{n=1}^{\infty}\frac{\ln n}{n}$ converge or diverge?
Solution: We will use integral test.

$$\begin{aligned}
\int_1^{\infty}\frac{\ln x}{x}dx &= \lim_{b\to\infty}\int_1^b \frac{\ln x}{x}dx \\
&= \lim_{b\to\infty}\int_0^{\ln b} u du \quad (\text{usub } u = \ln x) \\
&= \lim_{b\to\infty} 1/2u^2|_0^{\ln b} \\
&= \lim_{b\to\infty} 1/2(\ln b)^2 = \infty
\end{aligned}$$

Therefore, the series diverges.

(3) Does $\sum_{n=1}^{\infty}\frac{n}{n^2+1}$ converge or diverge?
Solution: Limit comparison test with $b_n = \frac{1}{n}$.

$$\lim_{n\to\infty}\frac{\frac{n}{n^2+1}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{n^2}{n^2+1} = 1$$

Thus we can compare. We know that $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges. Therefore by the limit comparison test the series diverges.

(4) Does $\sum_{n=1}^{\infty}\left(-\frac{3}{4}\right)^n$ converge or diverge?
Solution: This is a geometric series where $r = -\frac{3}{4} < 1$. Thus it is convergent. In fact, we know what it equals.

$$\sum_{n=1}^{\infty}\left(-\frac{3}{4}\right)^n = \frac{1}{1-(-\frac{3}{4})} = \frac{4}{7}$$

Classify the series as absolutely convergent, conditionally convergent, or divergent.

(1) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{10n+1}$

Solution: $\lim_{n\to\infty}(-1)^{n+1}\frac{n}{10n+1} \neq 0$. Thus, by the divergence test, the series diverges.

(2) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{5n}$

Solution: Let $f(n) = \frac{1}{5n}$. Then $f'(n) = -\frac{1}{5n^2} < 0$ for all n. Therefore, the sequences is decreasing. We know $f(n) > 0$. Thus by the alternating series does converge. However, $\displaystyle\sum_{n=1}^{\infty}\frac{1}{5n}$ is divergent. Thus $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{5n}$ converges conditionally.

(3) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^4}{e^n}$

Solution: Does $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^4}{e^n}$ converge? Yes, it does by the integral test. Thus the series converges absolutely.

(4) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos n}{n^2}$

Solution: Does $\sum_{n=1}^{\infty}\frac{|\cos n|}{n^2}$ converge? Let's compare with $b_n = \frac{1}{n^2}$. We know $\frac{|\cos n|}{n^2} \leq \frac{1}{n^2}$. Also, by p- test the series $\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges.

Therefore $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos n}{n^2}$ converges absolutely.