Name: ____

Math 8 Final Exam

December 3, 2011

Read all instructions carefully. No calculators are allowed. You may leave answers unevaluated; e.g., $6(\frac{1}{2})^5$. This is a closed book exam and no notes are allowed. You are not to provide or receive help from any outside source during the exam except that you may ask the instructor for clarification of a problem. You have three hours and you should attempt all 14 problems. There will be partial credit, so show all work.

(1) Let $f(x, y) = x^3 + y^2 - 3x - 4y$.

(a) Find and classify all the critical points of f.

(b) Find the absolute minimum and maximum of f on the triangle with vertices $(0, 0)$, $(0, 1)$, $(1, 0)$.

(2) Make all algebraic simplifications and substitutions (including u-substitutions) necessary to evaluate the following integrals. **You do not need to complete the integrals.**

(a) $\displaystyle\int \frac{1}{x^2\sqrt{x^2+9}}\,dx$

(b) $\displaystyle\int [\tan(x-1)\sec(x-1)]^6\,dx$

(c) $\displaystyle\int \frac{2x^2+4x-8}{x^3-4x}\,dx$

(3) Does the following series converge absolutely, converge conditionally, or diverge? State which test(s) you use.

$$\sum_{n=1}^{\infty} \left(\frac{-2n}{n+2} \right)^{5n}$$

(4) Let $f(x, y) = x^3 y^2$.

(a) Find the gradient of f at the point $(1, 1)$.

(b) Find an equation for the tangent plane to the surface $z = f(x, y)$ at the point $(1, 1, 1)$.

(c) Find $D_{\boldsymbol{u}} f(1, 1)$ for $\boldsymbol{u}$ a unit vector such that the angle between $\boldsymbol{u}$ and ∇f is $\pi/4$.

(5) Find the distance between the line $\langle 2t-1, 4t+3, -t\rangle$ and the plane $x-y-2z=12$.

(6) Find a power series representation, centered at 0, for

$$f(x) = \frac{4}{(1-2x)^2}.$$

Also find its radius and interval of convergence.

(7) Let $\vec{r}(t) = \langle \sin(2t), t+3, \cos(2t) \rangle$.

(a) What is the length of the curve traced out by $\vec{r}$ as t goes from 0 to 3?

(b) What is the curvature of $\vec{r}$ at $t = 1$?

(8) (a) What function $f(x)$ is given by the following Taylor series?

$$\sum_{n=0}^{\infty} \frac{2(-4)^n x^{2n+1}}{(2n+1)!}$$

(b) Give a bound for the error involved in using the Taylor polynomial of degree 5 to approximate f on $(-1, 1)$.

(9) Suppose $f(x, y, z) = x^2 + y^3 - xyz$, where $x = 2t - 1$, $y = t^2$, and $z = e^{-t}$. Find $\frac{df}{dt}$ at $t = 1$.

(10) Determine whether the following series converges, and if so, find the sum if possible.

$$\sum_{n=3}^{\infty} \frac{4^{n-1}}{5^{2n+2}}$$

(11) Find the following limit or show that it does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{y^4 - 4x^2}{2x^2 + y^2}$$

(12) Does the series converge or diverge? State which test(s) you use.

$$\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}}$$

(13) Determine whether the following statements are true or false. You need not show your work and there will be no partial credit.

(a) _____ The angle between a line and a plane it intersects is the angle between the line's direction vector and the plane's normal vector.

(b) _____ If $a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ is a vector in the plane $b_1x + b_2y + b_3z = 0$, then $a_1b_1 + a_2b_2 + a_3b_3 = 0$.

(c) _____ The cross product of two unit vectors is a unit vector.

(d) _____ For any nonzero and nonperpendicular vectors $\mathbf{u}$ and $\mathbf{v}$ with the angle θ between them, $\frac{|\mathbf{u}\times\mathbf{v}|}{\mathbf{u}\cdot\mathbf{v}} = \tan\theta$.

(e) _____ The sequence $\{(-1)^n(1-\frac{1}{n})\}$ is convergent.

(f) _____ The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+4}$ is convergent.