Name: _____ Section: _____

Math 8 Midterm 2

November 8, 2011

Read all instructions carefully. No calculators are allowed. You may leave answers unevaluated; e.g., $6(\frac{1}{2})^5$. This is a closed book exam and no notes are allowed. You are not to provide or receive help from any outside source during the exam except that you may ask the instructor for clarification of a problem. You have two hours and you should attempt all 9 problems. There will be partial credit, so show all work.

FERPA RELEASE: Because of privacy concerns, we are not allowed to return your graded exams in lecture without your permission. If you wish us to return your exam in lecture, please sign on the line indicated below. Otherwise, you will have to pick your exam up in your instructor's office after the exams have been returned in lecture.

SIGN HERE: _____

(1) (10 pts) Let $f(x) = x^3 \ln(1 + x^2)$. Find the Maclaurin series for f.

(2) (14 pts) Find the interval of convergence of the following power series.

$$\sum_{n=1}^{\infty} \frac{(x-5)^n}{5^n \sqrt{n}}$$

(3) Consider the following series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n 4^n x^n}{n!}$$

(a) (4 pts) What function is the series equal to?

(b) (10 pts) Suppose the 4th degree Taylor polynomial, $T_4(x)$, is used to approximate the series for $|x| < 1$. Give a bound on the error in using this approximation.

(4) Let O be the origin, P the point $(1, 2, 3)$, and Q the point $(0, -2, 2)$.

(a) (6 pts) Find the area of the parallelogram with P and Q the two vertices adjacent to vertex O.

(b) (6 pts) Find an equation for the plane containing O, P, and Q.

(5) Let a curve in 3-space be given by

$$\boldsymbol{r}(t) = \langle 4t,\ \sin(3t) + 2,\ \cos(3t) - 1 \rangle.$$

(a) (6 pts) Find the tangent line at $t = 0$.

(b) (6 pts) For what b is the length of the curve from $t = 0$ to $t = b$ equal to 10?

(6) (12 pts) Consider again the curve in 3-space given by

$$\boldsymbol{r}(t) = \langle 4t,\ \sin(3t) + 2,\ \cos(3t) - 1 \rangle.$$

Find the curvature.

(7) (10 pts) Find the distance between the line

$$x = t - 1, \quad y = 2t + 3, \quad z = 2 - t$$

and the point $(1, 3, -2)$.

(8) (8 pts) Find $\boldsymbol{r}(t)$ if $\boldsymbol{r}'(t) = \langle \cos(t), e^t, 2 \rangle$ and $\boldsymbol{r}(0) = \langle 1, 1, 1 \rangle$.

(9) (8 pts) Determine whether the following statements are true or false. You need not show your work and there will be no partial credit.

(a) _____ The only curves with constant curvature are straight lines and circles.

(b) _____ The lines $\langle 4t-1, 3-2t, 6t\rangle$ and $\langle 3-2t, t+1, 4-3t\rangle$ are skew.

(c) _____ The line $\langle 3t+2, t-1, 2t-5\rangle$ and the plane $x-y-z=0$ are parallel.

(d) _____ The angle between the planes $2x-y+3z=1$ and $x+y+z=20$ is $3\pi/4$.