Mathematics 8
Problems for Exam 1

The following problems were considered for the exam, but ultimately not included. This document is not indicative of the length or the distribution of problems on the actual exam.

1. A particle moving along the x-axis has position $x = 0$ at time $t = 0$, and at time t has velocity $v(t) = \dfrac{1}{\sqrt{1+t^2}}$.

   (a) Find the position of the particle at time t, for $t \geq 0$.

   (b) Find the average acceleration of the particle between times $t = 0$ and $t = 10$.

2. Find the volume of the solid obtained by rotating the area under the curve $y = \sin x$, for $0 \leq x \leq \pi$, about the x-axis.

3. Evaluate the following integrals

(a) $\displaystyle\int_0^{\infty} e^{-x} \sin x \; dx$

(b) $\displaystyle\int_0^{4} \frac{dx}{(9+x^2)^{3/2}}$

(c) $\displaystyle\int_0^{\sqrt{5}} \frac{x^3}{\sqrt{x^2+4}} dx$

4. Determine whether the following converge or diverge. Be sure to explain your reasoning.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n - 3}{n}$.

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln(n)}{\ln(n^2)}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n}\right)$

5. Evaluate the following. (Your answer should be a number, $+\infty$, $-\infty$, or "diverges" if it diverges but not to $+\infty$ or $-\infty$.) Be sure to explain your reasoning.

   (a) $\lim_{n\to\infty} \dfrac{\ln(n^3+5)}{n}$

   (b) $\lim_{n\to\infty} \dfrac{n\ln(n)}{n^2+5}$

   (c) $\lim_{n\to\infty} n^2(\cos(1/n)-1)$

6. Let $a_n = \dfrac{1}{(n+3)^3}$.

   Assume you want to use the partial sum $s_c = \displaystyle\sum_{n=1}^{c} a_n$ to approximate the value of $\displaystyle\sum_{n=1}^{\infty} a_n$.

   To guarantee an error of at most $\dfrac{8}{10^6}$, how large must c be?