1. Let $f(x, y) = 5 + 3x^2 + 3y^2 + 2y^3 + x^3$.

(a) Find all critical points of $f$.

(b) Use the second derivatives test to classify the critical points you found in (a) as a local maximum, local minimum, saddle point, or N/A if the test does not yield any information.

(c) Determine whether or not $f(x, y)$ has a global maximum in the xy-plane. Justify your answer.
2. Suppose that as you move away from the point $(2, 0, 1)$, the function $f(x, y, z)$ increases most rapidly in the direction of the vector $3\hat{i} - \hat{j} + 5\hat{k}$ and the rate of increase of $f$ in this direction is 7. What is $\nabla f(2, 0, 1)$?
3. Consider the function \( f(x, y) = x^3 + y^3 - 9xy + 1 \)

(a) Find and classify the critical points on \( f(x, y) \).

(b) Find the absolute maximum and minimum values of \( f(x, y) \) in the triangle with vertices \((0, 4), (4, 0),\) and \((4, 4)\).
4. (a) Find a set of parametric equations for the line passing through the point \((2, 1, -1)\) and normal to the tangent plane of

\[ 4x + y^2 + z^3 = 8 \]

(b) Suppose that \(z = f(x, y)\), where \(x = e^t\) and \(y = t^2 + 3t + 2\). Given that \(\partial z/\partial x = 2xy^2 - y\) and \(\partial z/\partial y = 2x^2y - x\), find \(dz/dt\) when \(t = 0\).

(c) Let \(f(x, y) = (x - y)^3 + 2xy + x^2 - y\). Find the linear approximation \(L(x, y)\) near the point \((1, 2)\).
5. Find the equation of the tangent plane to the surface \( z = 4x^3y^2 + 2y \) at the point \((1, -2, 12)\).
6. Find all points at which the surface

\[ x^2 + y^2 + 2x = z^2 \]

has a vertical tangent plane.
7. Suppose that $f(x, y)$ is the distance between the origin $(0, 0)$ and the point $(x, y)$. Use what you know about the significance of the gradient to find 

$$\nabla f(3, 4)$$

without finding a formula for $f$ or computing any derivatives.
8. Find a function parametrizing the curve $\gamma$ given by the intersection of the plane with equation $z = x - y$ and the cylinder with equation $x^2 + y^2 = 1$.

Find an equation for the line tangent to $\gamma$ at the point where $(x, y) = (0, 1)$.

Represent the arclength of $\gamma$ as an integral. You do NOT have to evaluate this integral; just write it down.