1. Determine the volume of the solid S that is obtained by rotating the region given by

$$0 \leq x \leq \pi \qquad \text{and} \qquad 0 \leq y \leq \sqrt{x \sin x}$$

around the x-axis.

2. Determine the center and radius of the sphere given by

$$x^2 + y^2 + z^2 - 4x - 2y + 4z = 0.$$

3. Determine the vector of length 10 that points in the opposite direction as $(0, 3, -4)$.

4. Determine parametric equations for the line that contains the point $(-5, 1, 4)$ and is perpendicular to the plane $2x + 3y - 6z + 7 = 0$.

5. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ denote vectors in $\mathbb{R}^3$. Which of the following expressions make sense? Write "**S**" if an expression makes sense, and "**N**" if it does not.

   Recall that

   - ☐ "$\times$" denotes the cross product,
   - ☐ "$\cdot$" denotes the dot product, and
   - ☐ concatenation denotes scalar multiplication.

   For example, if c is a scalar and $\mathbf{v}$ is a vector, then the concatenation "$c\mathbf{v}$" makes sense since it denotes scalar multiplication, whereas an expression like "$c + \mathbf{v}$" does not make sense.

   (a) ☐ $(\mathbf{u} + \mathbf{v})\mathbf{w}$

   (b) ☐ $(\mathbf{u} + \mathbf{v}) \times \mathbf{w}$

   (c) ☐ $(\mathbf{u} \cdot \mathbf{v}) \times \mathbf{w}$

   (d) ☐ $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$

   (e) ☐ $(\mathbf{u} \cdot \mathbf{v}) \cdot \mathbf{w}$

6. Which of the following statements are always true? Write "**T**" for true and "**F**" for false. Your computations will not be graded on this problem.

   If $\mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^3$, then

   (a) ☐ the projection of $\mathbf{v}$ onto $\mathbf{w}$ has length less than or equal to $\|\mathbf{v}\|$.

   (b) ☐ the projection of $\mathbf{v}$ onto $\mathbf{w}$ has length less than or equal to $\|\mathbf{w}\|$.

   (c) ☐ $|\mathbf{v} \cdot \mathbf{w}| \geq \|\mathbf{v}\| \, \|\mathbf{w}\|$.

   (d) ☐ $\|\mathbf{v} + \mathbf{w}\| \leq \|\mathbf{v}\| + \|\mathbf{w}\|$.

   (e) ☐ $\|\mathbf{v} \times \mathbf{w}\|^2 + |\mathbf{v} \cdot \mathbf{w}|^2 = \|\mathbf{v}\|^2 \|\mathbf{w}\|^2$.

7. Suppose that **a** and **b** are two vectors in $\mathbb{R}^3$ such that if **a** and **b** are drawn emanating from the origin, they both lie in the xy-plane, **a** in the third quadrant ($x < 0$ and $y < 0$) and **b** in the second quadrant ($x < 0$ and $y > 0$.)

   Suppose also that we know $\|\mathbf{a}\| = 1$ and $\|\mathbf{b}\| = 4$ as well as $\mathbf{a} \cdot \mathbf{b} = 2$.

   (a) Is the vector projection of **b** onto **a** longer than **a**, shorter than **a**, or the same length as **a**?

   (b) In what direction does $\mathbf{a} \times \mathbf{b}$ point? (Give a unit vector in this direction.)

   (c) Find the length of $\mathbf{a} \times \mathbf{b}$.

8. Consider the motion given by

$$\mathbf{r}(t) = (3t, \sin(4t), -\cos(4t)).$$

Compute

(a) the velocity at time t

(b) the speed at time t

(c) a unit vector $\mathbf{T}(t)$ in the direction of motion at time t

(d) the rate of change of $\mathbf{T}(t)$ with respect to time, $\mathbf{T}'(t)$

(e) the norm (magnitude) of $\mathbf{T}'(t)$

(f) the unit normal vector at time t

(g) the unit binormal vector at time t

(h) the distance travelled between times $t = 0$ and $t = 2$