Math 8: Calculus in one and several variables
Winter 2019 - Homework 7

Return date: Friday 02/22/19

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keywords: *tangent planes, chain rule in several variables*

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences and label any diagrams. Please show your work; no credit is given for solutions without work or justification.

exercise 1. *(3 points)* For the following functions find the indicated partial derivatives.

a) $f(x, y) = x^4 y - x^3 y^3$. Find f_{xxx} and f_{yxy}.

b) $f(x, y, z) = y \cdot e^{x^3+y^2+z^3}$. Find f_{yz}.

exercise 2. *(3 points)* Find an equation for the tangent plane to the graph of the given function at the given point.

a) $f(x, y) = x^2 + 3y^2$ at the point $P = (1, 1, 4)$. Sketch the function and the plane.

b) $f(x, y) = e^{x+2y}$ at the point $P = (-2, 1, 1)$.

exercise 3. *(3 points)* Find the linear approximation $L(x, y)$ or linearization of the function at the given point.

a) $f(x, y) = \sqrt{xy + 2}$ at the point $(x, y) = (4, 1)$.

b) $f(x, y) = \frac{x+2}{y-2}$ at the point $(x, y) = (0, 0)$.

Explain how you have obtained your answer.

exercise 4. *(4 points)* Suppose you need to know the tangent plane of a surface S containing the point $P = (2, 1, 3)$. You do not know the equation for S, but you know that the curves

$$\mathbf{r}_1(t) = \langle 2 + 3t, 1 - t^2, 3 - 4t + t^2 \rangle \quad \text{and} \quad \mathbf{r}_2(s) = \langle 1 + s^2, 2s^3 - 1, 2s + 1 \rangle$$

both lie in S and pass through P.
Find an equation of the tangent plane of S passing through P.

exercise 5. *(4 points)* Use the chain rule to find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ where

$$z = \left(x^2 + y^2\right)^3 \quad \text{and} \quad x = s \cdot \ln(t)\,,\ y = t \cdot e^s.$$

Hint: Use the chain rule to differentiate $(x^2 + y^2)^3$. Do not multiply it out.

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exercise 6. *(3 points)* The temperature at a point (x, y) in the plane is given by the function $T(x, y)$. A bug crawls along a path c, such that its position after time t in seconds is given by

$$c: \ \mathbf{r}(t) = \langle \sqrt{3-t}, 3t \rangle \quad \text{where} \quad t \in [0, 10].$$

From measurements you know that the temperature function satisfies

$$T_x(\sqrt{2}, 3) = 2 \quad \text{and} \quad T_y(\sqrt{2}, 3) = 3.$$

How fast is the temperature rising on the bugs path after 1 seconds?