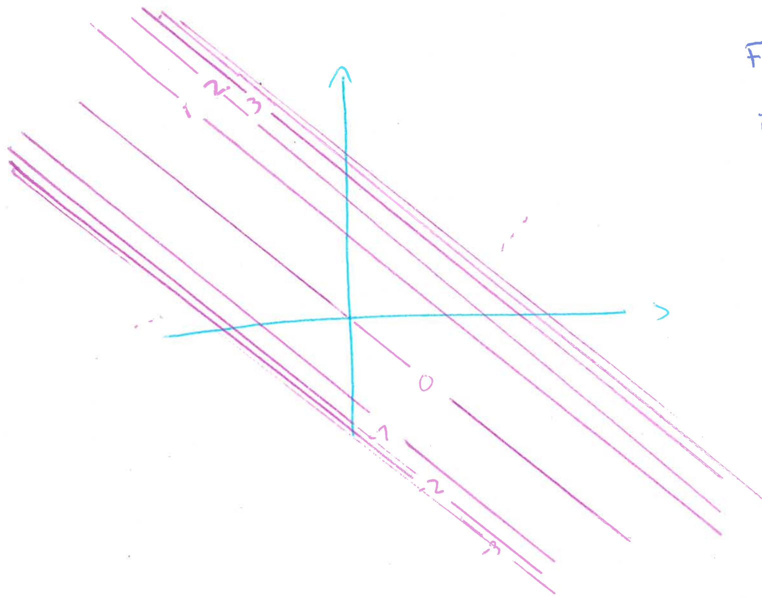
Math 8 - winter 2020

Answer key for homework due 2/24.

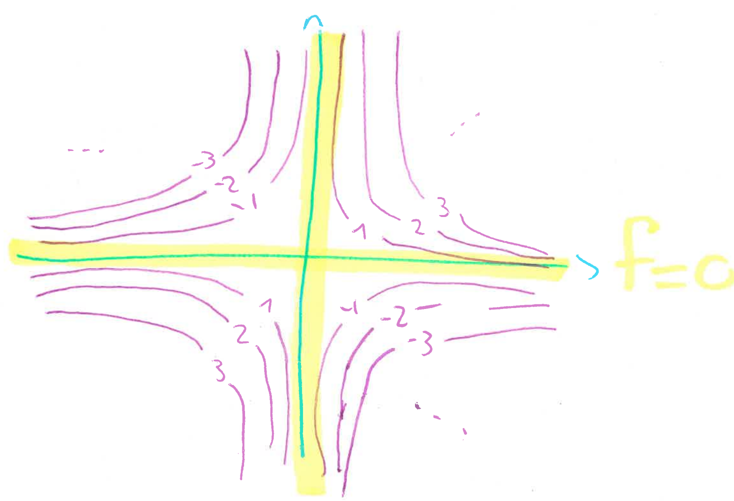
1. HW assigned 2/17.

#1 a) The level curves will be lines parallel to $y=-x$, not equally spaced. It should look like



For question 2, that is picture (iii)

b) The level curves are of the form $y=\frac{k}{x}$, for a fixed value of k, with also a cross along the x- and y-axes. That is

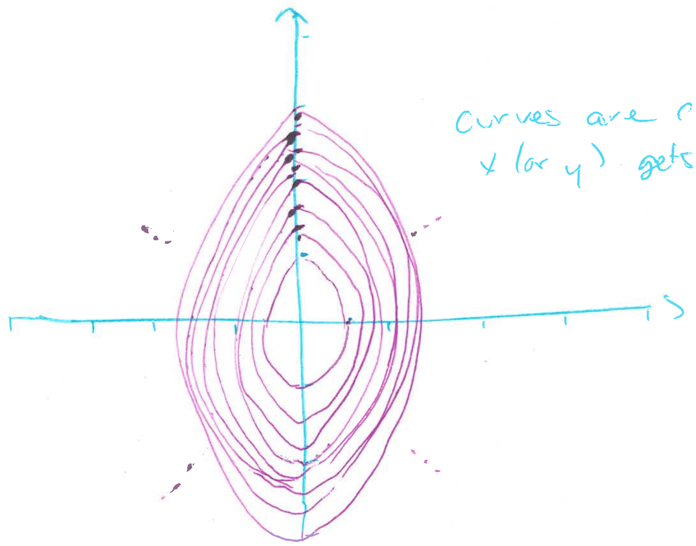


For question 2, that is picture (i).

(2)

(c) $f(x,y) = 4x^2 + y^2$

Since the graph of f is an elliptic parabolloid, the level curves will be ellipses of y-radius twice bigger than the x-radius. The ellipses will not be equally spaced.



That is picture (ii) for question 2.

2. Written HW assigned 2/19.

1. f is continuous at $(0,0)$ if $f(0,0) = \lim_{(x,y)\to(0,0)} f(x,y)$.

Here,

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{\sqrt{x^2+y^2}} = \lim_{r\to 0} \frac{r^2(\cos^2\theta - \sin^2\theta)}{r} = \lim_{r\to 0} r(\cos^2\theta - \sin^2\theta)$$

Switching to polar coordinates.

$$= 0 \quad (\text{as } r \to 0).$$

Hence f is continuous.

2. $\lim_{x\to 0} \frac{|x|}{x}$ does not exist, since approaching 0 from left to right, we get that the limit is $\lim_{x\to 0} \frac{-x}{x} = -1$, and approaching from the right, $\lim_{x\to 0} \frac{|x|}{x} = \lim_{x\to 0} \frac{x}{x} = 1$. Since they do not match, the limit does not exist.

3. $\lim_{(x,y)\to(0,0)} \frac{g(x,y)}{\sqrt{x^2+y^2}} = \lim_{(x,y)\to(0,0)} f(x,y)$, and we proved in part

1. that this is 0.

4. $\lim_{(x,y)\to(0,0)} \frac{f(x,y)}{\sqrt{x^2+y^2}} = \lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ does not exist:

Setting $x=0$, we get

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2} = \lim_{y\to0} \frac{-y^2}{y^2} = -1$$

Setting $y=x$, we get

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2} = \lim_{x\to0} \frac{x^2-x^2}{x^2+x^2} = \lim_{x\to0} \frac{0}{2x^2} = 0.$$

Since they do not match, the limit does not exist.

3. HW assigned 2/21

#1 we should expect $\frac{\partial f}{\partial t}$ to be negative, since $\frac{\partial f}{\partial t}$ is the rate of change of temperature over time. Since the block is warmer than the room at the beginning, the block should get colder over time.

#2. $\frac{\partial f}{\partial x}$ should be positive, since x is the depth in the block. Since the block is getting colder in that room, I expect the outside of the block to be colder than the inside, and, as travelling from the outside to the inside, the temperature should increase, that is $\frac{\partial f}{\partial x} > 0$.