Math 8
Winter 2020

Preliminary Homework
Assigned Monday, January 27

Note: Preliminary homework is always graded credit or no credit. **You get full credit for completing the assignment, whether or not your answers are correct, as long as your work shows you have thought about the problem.** The purpose of preliminary homework is to start you thinking about the topic of the next class.

You may use your preliminary homework for in-class activities with your classmates. You should be sure to think about these questions so you will be prepared.

Preliminary homework is always due at the *beginning* of the next class.

Suppose point P in $\mathbb{R}^3$ has coordinates (a, b, c).

We can think of this as meaning: To reach point P, start at the origin and go a units in the x-direction, then go b units in the y-direction, then go c units in the z-direction.

We can represent the motion from the origin to P by an arrow beginning at the origin and ending at P. This arrow represents a *vector*, which has the same coordinates as P, and is denoted $\langle a, b, c\rangle$. (We use angle brackets to indicate that these coordinates represent a vector rather than a point.)

This vector represents motion of a units in the x-direction, b units in the y-direction, and c units in the z-direction, no matter where we start. (It's like saying "go 15 yards due east.") So if we draw an arrow beginning at $(1, 1, 1)$ and ending at $(a + 1, b + 1, c + 1)$, that arrow represents the same vector $\langle a, b, c\rangle$, because in moving from $(1, 1, 1)$ to $(a+1, b+1, c+1)$ we also go a units in the x-direction, b units in the y-direction, and c units in the z-direction.

This is a small but important point. A vector in $\mathbb{R}^3$, algebraically, is just an ordered triple of numbers $\langle a, b, c\rangle$. We can draw this vector, or represent it geometrically, as an arrow beginning at any point (x, y, z) and ending at point $(x + a, y + b, z + c)$. Different arrows with the same direction and length are different pictures of the same vector.

1. For each pair of points, give the vector that represents motion from the first point to the second, and also give the length of that vector. (The length of the arrow from P to Q is, of course, the distance from P to Q.)

   (a) $P = (1, 1, 1)$ $Q = (1, 2, 3)$

   (b) $Q = (1, 2, 3)$ $R = (3, 3, 3)$

   (c) $P = (1, 1, 1)$ $R = (3, 3, 3)$

2. We are going to add vectors in such a way that if vector $\vec{v}$ represents motion from P to Q, and vector $\vec{w}$ represents motion from Q to R, then vector $\vec{v} + \vec{w}$ represents motion from P to R.

   Is the length of $\vec{v} + \vec{w}$ equal to the sum of the lengths of $\vec{v}$ and $\vec{w}$? How do you know?