Math 8
Winter 2020

Preliminary Homework
Assigned Wednesday, February 12

Note: Preliminary homework is always graded credit or no credit. **You get full credit for completing the assignment, whether or not your answers are correct.** The purpose of preliminary homework is to start you thinking about the topic of the next class.

You may use your preliminary homework in activities with your classmates. You should be sure to think about these questions so you will be prepared.

Preliminary homework is always due at the *beginning* of class.

**Assignment:** A particle travels around the unit circle in the $xy$-plane with position function at time $t \geq 0$ given by $\vec{r}(t) = \langle \cos \pi t^2, \sin \pi t^2 \rangle$. The object's velocity is given by $\vec{v}(t) = \vec{r}\,'(t)$, and its acceleration is given by $\vec{a}(t) = \vec{v}\,'(t)$. Note that velocity and acceleration are both vectors.

1. Find the object's velocity and acceleration at time $t$.

2. Let $f(t)$ denote the object's speed at time $t$. Find $f(t)$ and $f'(t)$.

3. For $t = \frac{1}{\sqrt{2}}$ and $t = 1$, write the object's acceleration $\vec{a}(t)$ at time $t$ as the sum of two parts, $\vec{a}_{tan}(t)$ tangent to the object's path and $\vec{a}_{norm}(t)$ normal to the object's path.

   (Hint: Draw pictures. Second hint: Don't try to do this in general; just do it for $t = \frac{1}{\sqrt{2}}$ and $t = 1$.)

4. Let $f(t)$ denote the object's speed at time $t$. Show that for $t = \frac{1}{\sqrt{2}}$ and $t = 1$,

$$|\vec{a}_{tan}(t)| = |f'(t)|.$$

   In other words, the magnitude of the part of acceleration in the direction of motion tells us how fast the object's speed is changing.

   (The part of acceleration $\vec{a}_{norm}(t)$ normal to the direction of motion is associated with the object's change in direction. Its magnitude is affected by both the object's speed and the curvature of the object's path. We will see this shortly.)

**Note:** As we see here, when an object is moving in two or three dimensions, the rate of change of speed with respect to time, $f'(t)$ in this problem, is *not* the same as acceleration. It is one component of acceleration.