

Math 8  
Winter 2020

Preliminary Homework  
Assigned Friday, January 10

Note: Preliminary homework is always graded credit or no credit. **You get full credit for completing the assignment, whether or not your answers are correct, as long as your work shows you have thought about the problem.** The purpose of preliminary homework is to start you thinking about the topic of the next class.

You may use your preliminary homework for in-class activities with your classmates. You should be sure to think about these questions so you will be prepared.

Preliminary homework is always due at the *beginning* of the next class.

**Exercises:**

The Taylor series for  $\ln x$  centered at 1, and evaluated at  $x = 2$ , gives the alternating harmonic series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots .$$

We know this series converges by the alternating series test.

In fact, it converges to the actual value of  $\ln 2$ .

1. The first few partial sums of this series are

$$S_0 = 0 \quad S_1 = 1 \quad S_2 = 1 - \frac{1}{2} = \frac{1}{2} \quad S_3 = 1 - \frac{1}{2} + \frac{1}{3} = \frac{5}{6} \quad S_4 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{7}{12}$$

Label these points  $S_0, S_1, \dots, S_4$  on a number line. (You will only need the interval between 0 and 1, so you can put 0 on the left end and 1 on the right end.)

Add to your picture the approximate locations of  $S_5$  and  $S_6$ .

Notice that in moving from  $S_n$  to  $S_{n+1}$  you move a smaller distance each time, alternating moves to the left and moves to the right.

2. Explain why the limit of the sequence of partial sums of this series must be between  $S_5$  and  $S_6$ .
3. What does this tell us about how big the difference  $S_5 - \ln 2$  could possibly be?
4. How big must you make  $n$  to guarantee the difference  $S_n - \ln 2$  is at most .01?