Math 8
Winter 2020

Written Homework Day 4
Assigned Monday, January 13

Note: Standard (not preliminary) written homework is graded on your work and your explanations, not just on your answer.

Explanations are important for many reasons. Being able to communicate what you know shows a depth of understanding beyond that of being able to get the right answer to a problem. Doing the mental work of putting explanations into words helps create that depth of understanding. On exams, we will grade your work and not just your answers, so this is good practice for taking exams.

For all these reasons, be sure to: show all your work; explain your reasoning; use clear English; write neatly so all this effort does not go to waste.

Written homework is always due at 10:00 AM on the following Monday.

Homework: The Maclaurin series for $ln(1+x)$ converges to the value $ln(1+x)$ whenever $-1 < x \leq 1$:

$$\sum_{k=1}^{\infty}(-1)^{k+1}\left(\frac{x^k}{k}\right) = ln(1+x).$$

We can use this to approximate $\ln(1+x)$:

$$\ln(1+x) \approx \sum_{k=1}^{n}(-1)^{k+1}\left(\frac{x^k}{k}\right).$$

The error in the approximation is the difference between the approximation and the actual value. For $-1 < x \leq 1$, where the Maclaurin series gives the actual value,

$$error = \left|\ln(1+x) - \sum_{k=1}^{n}(-1)^{k+1}\left(\frac{x^k}{k}\right)\right| = \left|\sum_{k=1}^{\infty}(-1)^{k+1}\left(\frac{x^k}{k}\right) - \sum_{k=1}^{n}(-1)^{k+1}\left(\frac{x^k}{k}\right)\right| =$$

$$\left|\sum_{k=n+1}^{\infty}(-1)^{k+1}\left(\frac{x^k}{k}\right)\right|.$$

1. Find a value of n for which the error in this approximation to $\ln(2)$, or $\ln(1+1)$, is at most .01. Hint: The sum that gives the error is alternating.

2. Find a value of n for which the error in this approximation to $\ln(.5)$, or $\ln(1+(-.5))$, is at most .01. Hint: Try comparing the sum that gives the error to a geometric series.