Math 8
Winter 2020

Written Homework Day 5
Assigned Wednesday, January 15

Note: Standard (not preliminary) written homework is graded on your work and your explanations, not just on your answer.

Explanations are important for many reasons. Being able to communicate what you know shows a depth of understanding beyond that of being able to get the right answer to a problem. Doing the mental work of putting explanations into words helps create that depth of understanding. On exams, we will grade your work and not just your answers, so this is good practice for taking exams.

For all these reasons, be sure to: show all your work; explain your reasoning; use clear English; write neatly so all this effort does not go to waste.

Written homework is always due at 10:00 AM on the following Monday.

1. We already showed that the Maclaurin series for $g(x) = \dfrac{1}{1-x}$, which is a geometric series, has radius of convergence $R = 1$, and inside that radius of convergence (for $|x| < 1$) it converges to $g(x)$. That is, for $|x| < 1$, we have:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n.$$

   Make a substitution $x = 1-u$ in this equation, and manipulate the result as necessary, to get an equation of the form

$$\frac{1}{u} = \sum_{n=0}^{\infty} c_n (u-1)^n.$$

   For what values of u is this valid?

2. Integrate both sides of the equation you found in (a) (integrating the series term-by-term), and manipulate the result as necessary, to get an equation of the form

$$\ln(x) = \int_1^x \frac{1}{u}\, du = \sum_{n=1}^{\infty} d_n (x-1)^n.$$

   For what values of x does the theorem about integrating and differentiating power series guarantee that this is valid?

3. It turns out that for $x = 2$ the series you found in part 1 converges to $\ln(2)$.

   What do you think happens for $x = 0$, and why? (Any plausible answer gets credit for this part. You do not have to guess correctly.)