Math 8
Winter 2020

Written Homework Day 6
Assigned Friday, January 17

Note: Standard (not preliminary) written homework is graded on your work and your explanations, not just on your answer.

Explanations are important for many reasons. Being able to communicate what you know shows a depth of understanding beyond that of being able to get the right answer to a problem. Doing the mental work of putting explanations into words helps create that depth of understanding. On exams, we will grade your work and not just your answers, so this is good practice for taking exams.

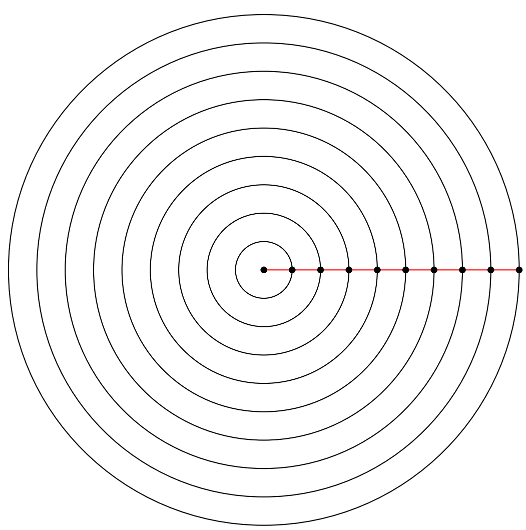
For all these reasons, be sure to: show all your work; explain your reasoning; use clear English; write neatly so all this effort does not go to waste.

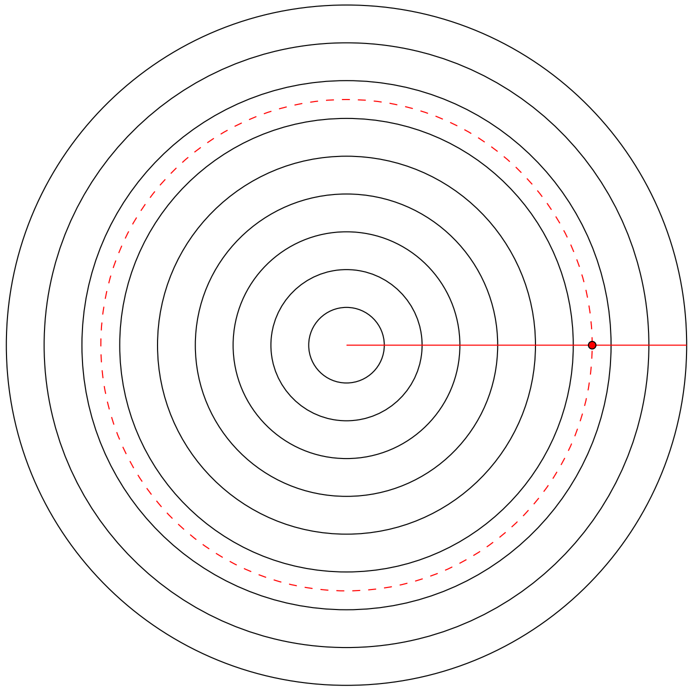
Written homework is always due at 10:00 AM on the following Monday.

Suppose you are a mathematician of the ancient world. You have defined π to be the ratio of a circle's circumference to its diameter, so you know you can express the circumference C of a circle in terms of its radius r as $C = 2\pi r$.

Now you would like to approximate the area of a disc of radius r. You begin by slicing up the disc into thin concentric rings, as follows:

Draw a line segment from the center of the disc to some point on the edge (shown in red in the picture). We can think of this as a portion of the x-axis, with $x = 0$ at the center and $x = r$ at the edge. Divide this line segment into n-many small segments of length Δx. Through each of the division points, draw a circle with the same center as the center of the disc.





1. Choose a point in the i^{th} small segment a distance x_i from the center of the disc. The thin ring corresponding to this segment (the i^{th} ring out from the center) is approximately the same as a rectangular strip of

   length equal to ______________________ and width equal to ______________________.

2. The area of the i^{th} ring is approximately: $A_i \approx$

3. Approximate the area of the disc as a sum.

   $A \approx$

4. Now, as a mathematics student of the modern world, express the limit of your approximate area, as $n \to \infty$, as a definite integral.

   $A =$

5. Evaluate the integral to find the area A of the disc. Your answer should give you the formula you know.

   $A =$

(Note that the details in this problem are not at all historically accurate. If you were a mathematician of the ancient world approximating the area of a disc, you would use the easier method of dividing the disc into skinny wedges, like cutting a pie. Mathematicians of ancient Greece and China did, however, use the method of solving a problem by breaking it up into smaller and smaller pieces. They didn't, of course, have the tools of the modern calculus to help them.)