Math 8
Winter 2020

Preliminary Homework
Due Wednesday, January 22

Note: Preliminary homework is always graded credit or no credit. **You get full credit for completing the assignment, whether or not your answers are correct, as long as your work shows you have thought about the problem.** The purpose of preliminary homework is to start you thinking about the topic of the next class.

You may use your preliminary homework for in-class activities with your classmates. You should be sure to think about these questions so you will be prepared.

Preliminary homework is always due at the *beginning* of the next class.

In these problems, we will find the formula for the volume of a cone of height h and base radius r.

1. If you slice the cone with a plane parallel to its base at a distance of x units below the top point, that slice is a disc, and the portion of the cone above the plane is a smaller cone.

   The height of the smaller cone is x. What is its base radius?

   Hint: The smaller cone is similar to the original cone. The word similar here means the same thing it means in high school geometry.

2. Draw the x-axis passing through the point of the cone and the center of its base, with $x = 0$ at the top of the cone and $x = h$ at the base of the cone. Break the interval $[0, h]$ into n-many subintervals of length Δx, and slice the cone with planes parallel to the base of the cone at each division point of the subintervals. This slices the cone into a bunch of slabs, one for each subinterval.

   Choose x_i^* in the i^{th} subinterval. The i^{th} slab is almost the shape of a coin (a really, really short cylinder) with:

   (a) Thickness equal to ________________________;

   (b) Base radius equal to ________________________;

   (c) Volume equal to ________________________.

3. The volume V_i of the i^{th} slab is approximately equal to the volume of the coin shape from part 2. We can approximate the volume of the cone as the sum of the approximate volumes of the different slabs:

$$V = \sum_{n=1}^{\infty} V_i \approx \sum_{n=1}^{\infty} \underline{\hspace{6cm}}.$$

4. Use part 3 to express the volume of the cone as an integral.
5. Evaluate the integral from part 4 to find the volume of the cone.