Math 8
Winter 2020

Preliminary Homework
Assigned Wednesday, January 22

Note: Preliminary homework is always graded credit or no credit. **You get full credit for completing the assignment, whether or not your answers are correct, as long as your work shows you have thought about the problem.** The purpose of preliminary homework is to start you thinking about the topic of the next class.

You may use your preliminary homework for in-class activities with your classmates. You should be sure to think about these questions so you will be prepared.

NOTE: This homework is due at the beginning of Friday's class, whether or not your section has a x-hour on Thursday.

If a force of magnitude F acts on a moving object, in the same direction the object is moving, the *work* done on the object by that force is the magnitude F of the force times the distance d the object moves.

For example, near the earth's surface, the force exerted by gravity on an object of mass m has magnitude mg (where g is a constant) and the direction in which it acts is straight down. If an object of mass m falls straight down for a distance d, the work done on that object by gravity is the product $(mg)(d)$.

If you have taken physics, this will be familiar to you. If not, you should just accept that this is an important concept; the amount of work done is related to the amount of energy expended. Whether or not you have taken physics, you may know that this is an approximation.

For this particular assignment, you don't have to worry about units (such as feet or meters). We will consider them later.

Homework: A spherical planet has radius r. A bit of space debris located a distance $d > r$ from the center of the planet is acted on by a gravitational force of magnitude $\dfrac{A}{d^2}$, acting in the direction of the planet, where A is a constant.

This bit of space debris falls from a distance R from the center of the planet ($R > r$) to the surface of the planet.

1. View the x-axis as passing through the piece of space debris and the planet's center, with the initial position of the debris at $x = 0$ and the planet's center at $x = R$, so the debris moves along the x-axis as it falls. (See the picture on the next page.)

   What is the magnitude of the gravitational force acting on the debris at its initial position a distance R from the center of the planet? What is the value of x at the point the debris hits the planet's surface?

2. Divide the path of the debris into n-many small segments of length Δx. If the point x_i^* on the x-axis is in the i^{th} segment, approximately what is the gravitational force acting on the debris as it travels over that short segment of its path? Approximately how much work does the gravitational force do as the debris travels over that short segment?

3. Approximate the total amount of work done on the debris by the gravitational force, as the debris falls from its initial position to the surface of the planet, by a sum.

4. By taking a limit as the number n of segments approaches infinity, find an integral that represents the total amount of work done on the debris by the gravitational force, as the debris falls from its initial position to the surface of the planet. (You do not have to evaluate that integral.)