Math 8
Winter 2020

WeBWorK Day 11 Problem 3
Sample Solution

Problem: Let $\vec{a} = \langle 3, -2, 1\rangle$ and $\vec{b} = \langle 3, 3, -2\rangle$. Show that there are scalars s and t such that $s\vec{a} + t\vec{b} = \langle -3, 12, -7\rangle$.

We want to find s and t satisfying

$$s\langle 3, -2, 1\rangle + t\langle 3, 3, -2\rangle = \langle -3, 12, -7\rangle.$$

Working out the scalar multiplication and vector addition, this becomes

$$\langle 3s + 3t, -2s + 3t, s - 2t\rangle = \langle -3, 12, -7\rangle.$$

We can rewrite this as three scalar equations, one for each coordinate,

$$3s + 3t = -3$$

$$-2s + 3t = 12$$

$$s - 2t = -7.$$

Now we solve this system of equations. Subtracting the second equation from the first gives

$$5s = -15 \qquad s = -3.$$

Substituting into the third equation gives

$$-3 - 2t = -7 \qquad t = 2.$$

Now we check that this works. (A system of three equations in only two unknowns usually does not have a solution.)

$$s\vec{a} + t\vec{b} = -3\langle 3, -2, 1\rangle + 2\langle 3, 3, -2\rangle = \langle -9 + 6, 6 + 6, -3 - 4\rangle = \langle -3, 12, -7\rangle.$$

This is what we needed to show.

Note: If you start at the origin, move with displacement $s\vec{a}$ (in a direction parallel to $\vec{a}$), and then with displacement $t\vec{b}$, you will always end up on the plane containing the origin and parallel to vectors $\vec{a}$ and $\vec{b}$. That is, all the points with position vectors of the form $s\vec{a} + t\vec{b}$ lie on a plane. Three points on that plane are $(0, 0, 0)$, $(3, -2, 1)$, and $(3, 3, -2)$ This problem shows that the point $(-3, 12, -7)$ also lies on that plane.