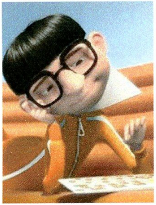
Vectors

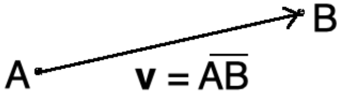


VECTOR: I'm applying for a villain loan. I go by Vector. It's a mathematical term, represented by an arrow with both direction and magnitude. Vector! That's me, because I commit crimes with both direction and magnitude. Oh yeah!

From "Despicable Me"

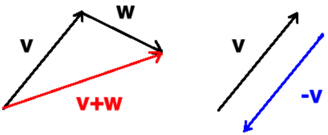
A **vector** is a quantity that has both magnitude and direction.

A geometric approach



Given two points A and B, the vector $\mathbf{v} = \vec{v} = \overrightarrow{AB} = \overline{AB}$ is the vector with initial point A, points in the direction of B (from A) and has magnitude equal to the length of the line segment $|AB|$.

Algebra of geometric vectors



Adding

Given two vectors $\vec{v}$ and $\vec{w}$, the vector $\vec{v} + \vec{w}$ is the vector with the same initial point as $\vec{v}$ and the same terminal point as $\vec{w}$.

Subtracting

Given two vectors $\vec{v}$ and $\vec{w}$, then the vector $-\vec{v}$ is the vector with the same magnitude as $\vec{v}$, but with opposite direction to $\vec{v}$ and $\vec{w} - \vec{v} = \vec{w} + (-\vec{v})$.

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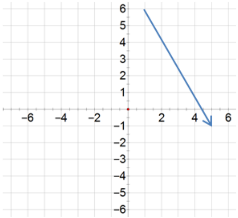
Scalar multiplication

If c is a scalar and $\vec{v}$ a vector, then the scalar multiple $c\vec{v}$ is the vector with with magnitude $|c|$ times the magnitude of $\vec{v}$, and with the same direction as $\vec{v}$ if $c > 0$ and opposite direction if $c < 0$. If $c = 0$ we get the **zero vector** $\vec{0}$.

An algebraic approach



Given two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, then $\overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$. The coordinates of $\overrightarrow{AB}$ are called the **components** of $\overrightarrow{AB}$

A **position vector** is a representation of a vector with its initial point at the origin.

The magnitude of $\vec{v} = \langle v_1, v_2, v_3 \rangle$ is $|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$.

Algebra of component vectors

Let $\vec{v} = \langle v_1, v_2, v_3 \rangle$ and $\vec{w} = \langle w_1, w_2, w_3 \rangle$.

Adding

$\vec{v} + \vec{w} = \langle v_1 + w_1, v_2 + w_2, v_3 + w_3 \rangle$
$\vec{v} - \vec{w} = \langle v_1 - w_1, v_2 - w_2, v_3 - w_3 \rangle$
$c\vec{v} = \langle cv_1, cv_2, cv_3 \rangle$

A **unit vector** is a vector with length 1.

Three special vectors

$\vec{i} = \langle 1, 0, 0 \rangle$, $\vec{j} = \langle 0, 1, 0 \rangle$ and $\vec{k} = \langle 0, 0, 1 \rangle$