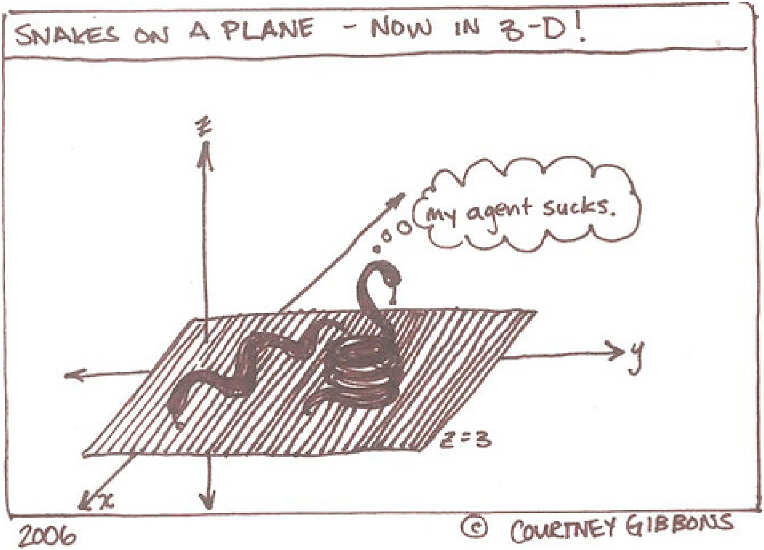
Lines and planes in $\mathbb{R}^3$



Equations of straight lines in $\mathbb{R}^3$

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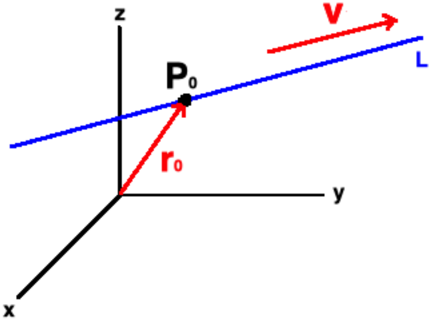
(i) any point $P_0(x_0, y_0, z_0)$ on L, and
(ii) any vector $\vec{v} =< a, b, c >$ parallel to L



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Then

- $\vec{r} = \vec{r}_0 + t\vec{v} = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$ (vector eqn of L)
- $x = x_0 + ta$, $y = y_0 + tb$ and $z = z_0 + tc$ (parametric eqns of L)
- $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$ (symmetric eqns of L)

Line segment from $P(p_1, p_2, p_3)$ to $Q(q_1, q_2, q_3)$:

$$\vec{r}(t) = (1 - t)\vec{p} + t\vec{q}, \qquad 0 \leq t \leq 1.$$

Equations of planes

Suppose P is a plane in $\mathbb{R}^3$. To find the equation of P we need:
(i) any point $P_0(x_0, y_0, z_0)$ on P, and
(ii) any normal (perpendicular) vector $\vec{n} =< a, b, c >$ to P

Set $\vec{r}_0 = \overrightarrow{OP_0}$, then

- $\vec{n} \cdot (\vec{r} - \vec{r_0}) = 0$ (vector equation)
- $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$ (scalar equation of L)
- $ax + by + cz + d = 0$ (linear equation)

Examples

1. (a) Find the 3 types of equations of the line L that passes through the points $(-2, 1, 1)$ and $(-4, -3, 1)$.
   Solution: $\vec{r}(t) =< -4, -3, 1 > +t < 2, 4, 0 >$
   (b) Find two other points on L. Sub any two values for t
   (c) Where does L intersect the zy-plane? $(0, 5, 1)$
   (d) Where does L intersect the xy-plane? It doesn't
2. Find the linear equation of the plane through $(6, -3, 4)$ with normal vector $< 2, -2, -1 >$. $2x - 2y - z = 14$
3. Find the linear equation of the plane through the (non-collinear) points $(1, 2, 3)$, $(-3, -2, 1)$ and $(0, 1, 0)$. $x - y = -1$
4. Find the equation of the line where the planes $x - y + 2z = 2$ and $x + 2y + 3z = 0$ intersect. $\vec{r}(t) =< \frac{4}{3}, -\frac{2}{3}, 0 > +t < -\frac{7}{3}, -\frac{1}{3}, 1 >$
5. Find the point where the following lines intersect or show that they are either parallel or skew: $\vec{r}_1(t) =< -1 + 2t, 2 - t, 4 - 3t >$ and $\vec{r}_2(s) =< 3 + 2s, 4 + 3s, 5 + 4s >$. (1,1,1)