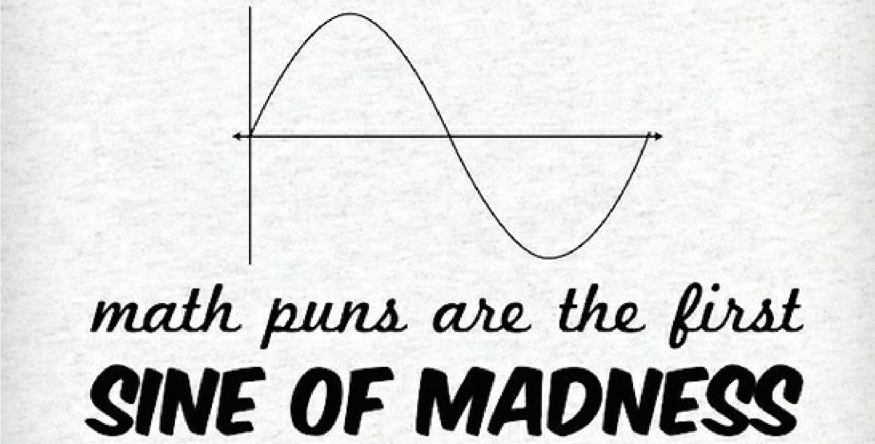
Vector Functions and Space Curves



Draw the following surfaces in $\mathbb{R}^3$:

- $z = x^2 + y^2$ (elliptic paraboloid)
- $z^2 = x^2 + y^2$ (cone)
- $z = x^2 - y^2$ (hyperbolic paraboloid)
- $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ (ellipsoid)
- $y = x^2$ (parabolic cylinder)

Vector valued $\vec{r} : \mathbb{R} \to \mathbb{R}^3$:

$$\vec{r} : t \mapsto < f(t), g(t), h(t) >$$

(identifying points in $\mathbb{R}^3$ with 3-component vectors)

1. Think of straight lines, e.g.

$$\begin{aligned} \vec{r}(t) &= < 2, 4, 5 > + t < 1, -2, 6 > \\ &= < 2 + t, 4 - 2t, 5 + 6t > \\ &= < f(t), g(t), h(t) > . \end{aligned}$$

2. $\vec{r}(t) = \langle \cos(t), \sin(t), 0 \rangle$ (Circle in xy-plane, but with a direction!)

Limits and continuity

Limit

If $\vec{r}(t) =< f(t), g(t), h(t) >$, then

$$\lim_{t\to a} \vec{r}(t) = \left\langle \lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t) \right\rangle,$$

provided **each** component-limit exists.

Continuity

$\vec{r}(t) =< f(t), g(t), h(t) >$ is continuous at $t = a$ if and only if

$$\lim_{t\to a} \vec{r}(t) = \vec{r}(a)$$

(i.e. if and only if **each component function is continuous**)

A **curve** γ is the range of a continuous vector function.

Examples

1. Sketch the curve whose equation is given by $\vec{r}(t) = \langle 2\cos(t), 2\sin(t), t \rangle$. ▸ Link
2. Find a vector function (i.e. a parametrization) for the ellipse $9x^2 + y^2 = 9$ in $\mathbb{R}^2$.
3. Find a parametrization for the curve of intersection of the cylinder $x^2 + y^2 = 4$ and the plane $z + 2y = 4$.
4. Find a parametrization for the curve of intersection of the sphere $x^2 + y^2 + z^2 = 4$ and the plane $x = 1$.