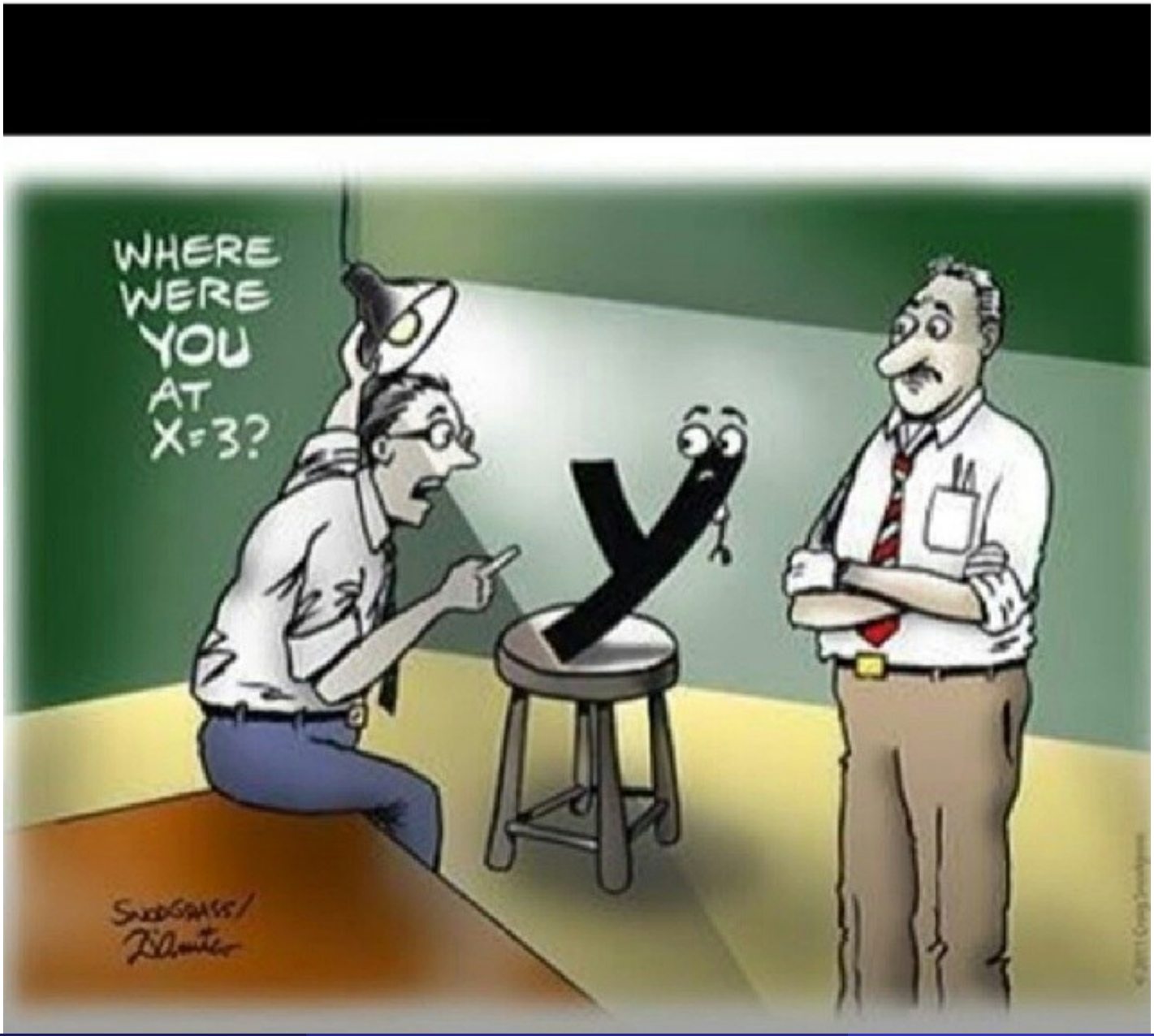
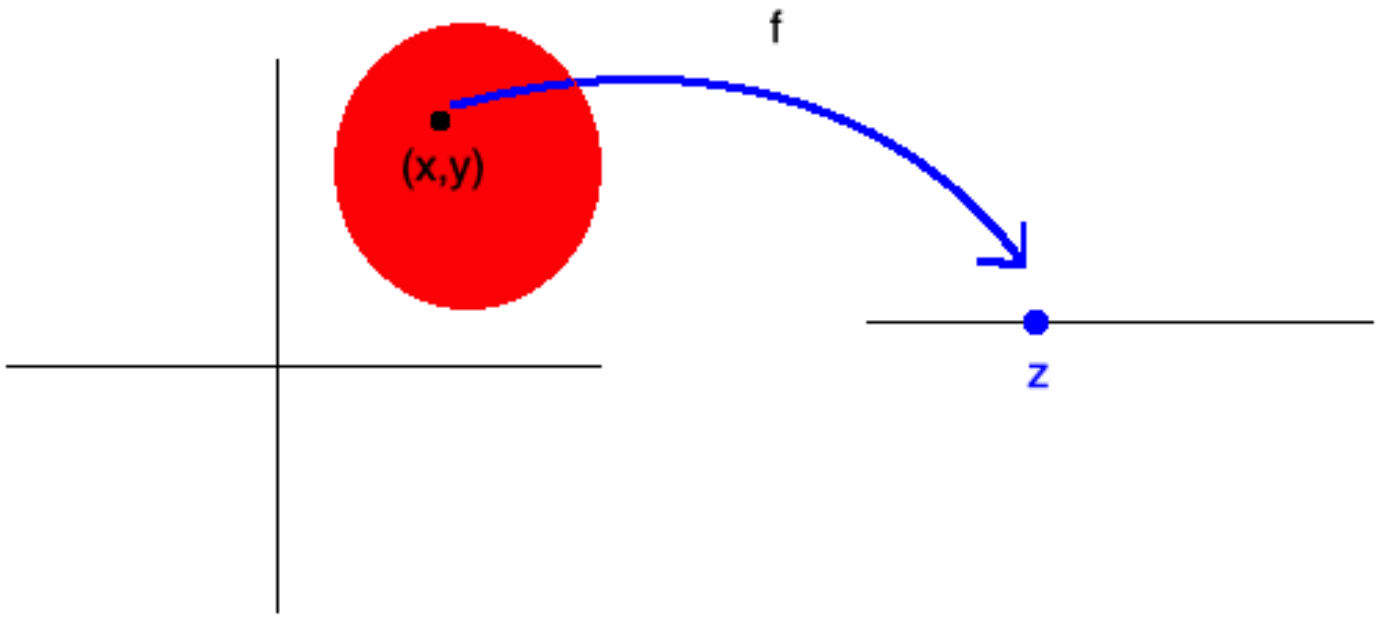
Functions of several variables - graphs and level curves



Function of two variables

A function f of two variables is a rule that assigns to an ordered pair $(x, y) \in \mathbb{R}^2$ a value $z \in \mathbb{R}$.



Function of two variables

1. More succinctly: a function f of two variables is function $f : D \to \mathbb{R}$, where $D \subseteq \mathbb{R}^2$ is the domain of f.
2. We write $z = f(x, y)$ (analogous to single variables: $y = f(x)$)

Example

1. $f(x, y) = \dfrac{\cos(x) + y^2}{xy}$.

   $f(\pi, 4) = \dfrac{\cos(\pi) + 4^2}{\pi(4)} = \dfrac{15}{4\pi}$
2. $g(x, y) = \sqrt{x} + 2\ln(2 - y) - \frac{1}{x}$.
   The domain of g is $\{(x, y) \in \mathbb{R}^2 : x > 0, y < 2\}$

Two-variable functions can be viewed as (1) tables of values (see Examples 2 and 3 in Stewward), (2) explicit formula or (3) graphically.

Graphs

Definition

The graph of a function $f : \mathbb{R}^2 \to \mathbb{R}$ with domain D is the set

$$\{(x, y, z) \in \mathbb{R}^3 : z = f(x, y) \text{ for every } (x, y) \in D)\}$$

Such graphs are typically surfaces in $\mathbb{R}^3$

Sketch

1. $f(x, y) = -x - 3y + 2$ (linear equation, i.e. a plane),
2. $f(x, y) = \sqrt{16 - x^2 - y^2}$ (hemisphere),
3. $f(x, y) = \sqrt{x^2 + y^2}$ (top half of a cone)
4. $f(x, y) = x^2 + y^2$ (elliptic paraboloid)

Level curves

Definition

The level curves of a function $f : \mathbb{R}^2 \to \mathbb{R}$ are the curves with equations $f(x, y) = k$, where k is a constant in the range of f.

Visual exmple

Level curves

Definition

The level curves of a function $f : \mathbb{R}^2 \to \mathbb{R}$ are the curves with equations $f(x, y) = k$, where k is a constant in the range of f.

Sketch the level curves of

1. $f(x, y) = -3x - y + 2$ for $k = 1, 2, 3$
2. $f(x, y) = \sqrt{16 - x^2 - y^2}$ for $k = 0, 7, 15$
3. $f(x, y) = \sqrt{x^2 + y^2}$ for $k = 1, 2, 3$

Examples

1. Find and sketch the domain of $f(x, y) = \sqrt{x^2 + y^2 - 4}$
2. Sketch the graph of $f(x, y) = 2 - x^2 - y^2$
3. Sketch the graph of $f(x, y) = \sqrt{4x^2 + y^2}$
4. Draw a contour map for $f(x, y) = ln(x^2 + 4y^2)$ showing several level curves