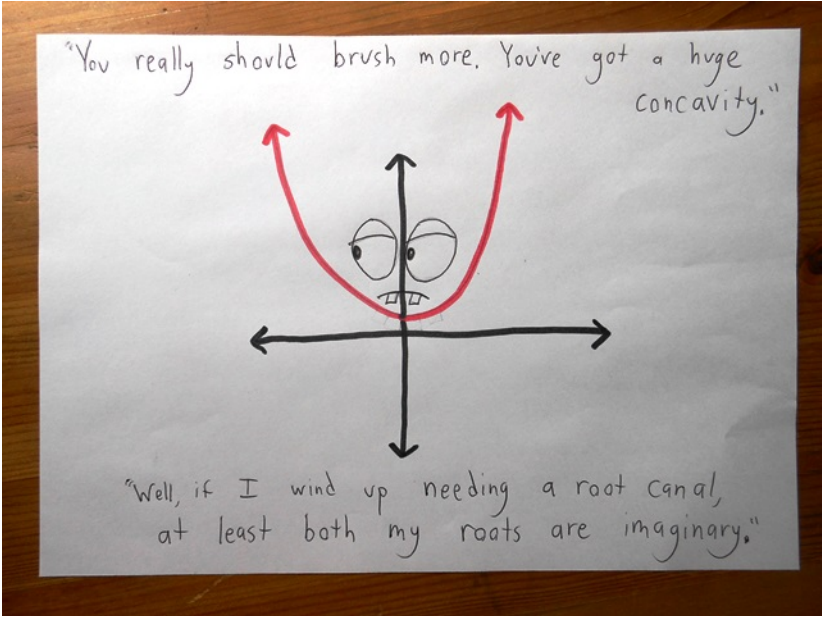
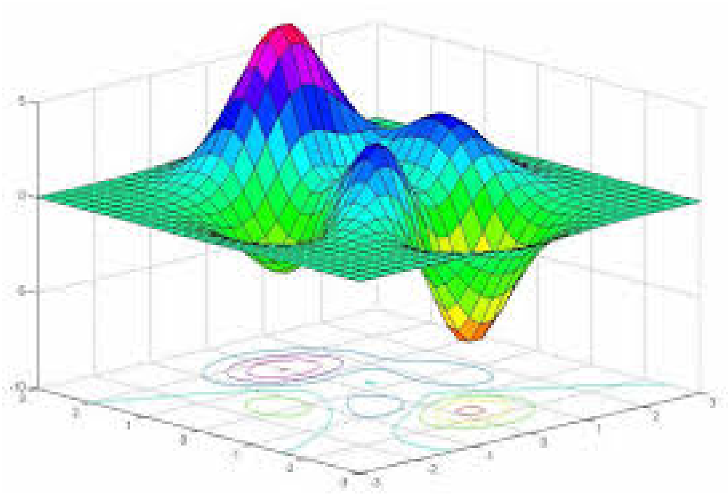
Maxima and Minima

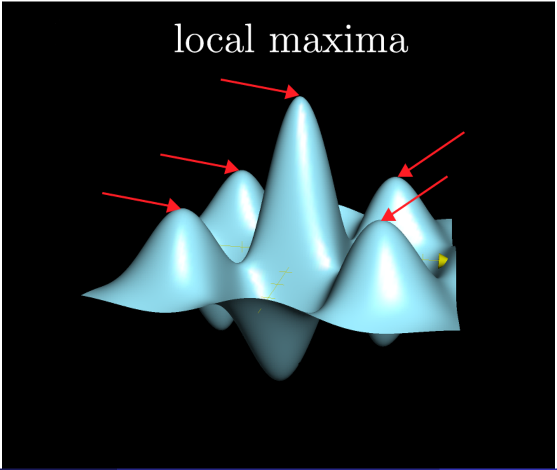




Definitions

Let $f(x, y)$ be a two variable function. Then f

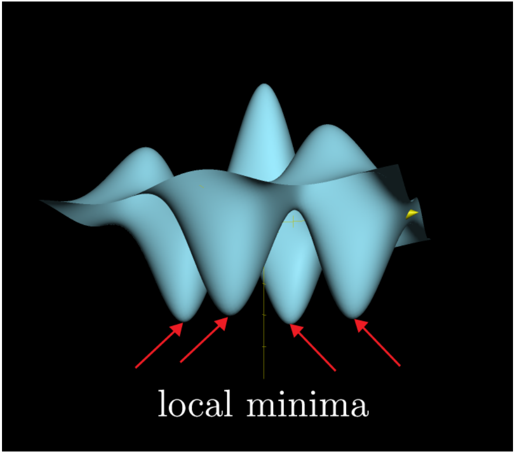
1. has a **local maximum** at (a, b) if $f(x, y) \leq f(a, b)$ for every (x, y) in some disk containing (a, b), and $f(a, b)$ is the **local maximum value**,



Definitions

Let $f(x,y)$ be a two variable function. Then f

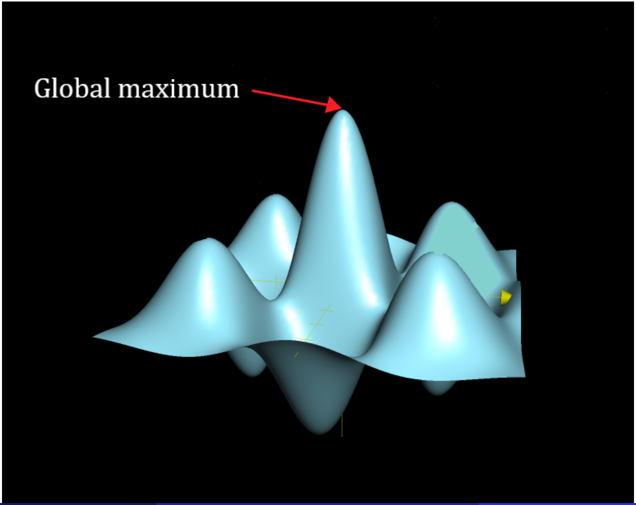
1. has a **local minimum** at (a,b) if $f(x,y) \geq f(a,b)$ for every (x,y) in some disk containing (a,b), and $f(a,b)$ is the **local minimum value**,



Definitions

Let $f(x, y)$ be a two variable function. Then f

1. has an **absolute maximum** at (a, b) if $f(a, b) \geq f(x, y)$ for **all** (x, y) (in the domain of f).



Definitions

Let $f(x, y)$ be a two variable function. Then f

1. has a **local minimum** at (a, b) if $f(x, y) \geq f(a, b)$ for every (x, y) in some disk with centre (a, b) and $f(a, b)$ is the **local minimum value**,
2. has a **local maximum** at (a, b) if $f(x, y) \leq f(a, b)$ for every (x, y) in some disk with centre (a, b) and $f(a, b)$ is the **local maximum value**,
3. has an **absolute minimum** at (a, b) if $f(a, b) \leq f(x, y)$ for **all** (x, y) (in the domain of f), and
4. has an **absolute maximum** at (a, b) if $f(a, b) \geq f(x, y)$ for **all** (x, y) (in the domain of f).

Theorem

If f has a local maximum or minimum at (a, b) and $f_x(a, b)$ and $f_y(a, b)$ both exist, then

$$f_x(a, b) = f_y(a, b) = 0.$$

Definition

A point (a, b) in the domain of f is a **critical point** (or stationary point)

1. if $f_x(a, b) = f_y(a, b) = 0$, or
2. if one of $f_x(a, b)$ or $f_y(a, b)$ does not exist.

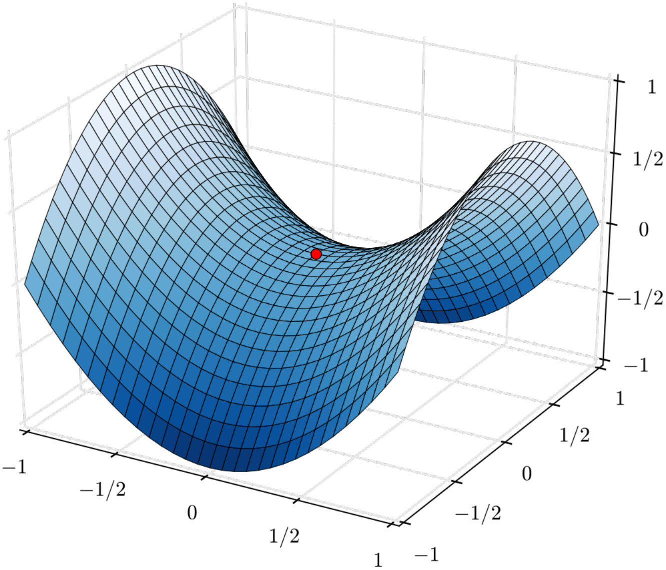
Second derivative test

Suppose f has continuous second order partial derivatives on a disk with centre (a, b) and that $f_x(a, b) = f_y(a, b) = 0$. Define

$$D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2 .$$

1. If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is local minimum.
2. If $D(a, b) > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is local maximum.
3. If $D(a, b) < 0$, then $f(a, b)$ is not a local maximum or minimum (in this case we call (a, b) a **saddle point**).

A saddle point:



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3. If $D(a, b) < 0$, then $f(a, b)$ is not a local maximum or minimum (in this case we call (a, b) a **saddle point**).

$D(a, b) = 0$ gives us no information, either way!

Absolute extrema

Extreme value theorem

If f is continuous on a closed and bounded set $D \subset \mathbb{R}$, the f attains an absolute maximum and minimum in D.