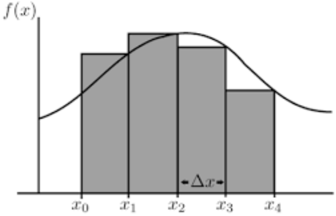
Riemann Sums

Recap



- Divide an interval $[a, b]$ into n subintervals $\Delta x = \frac{b-a}{n}$.
- Choose a point x_i^* in interval i for $i = 1, 2, \ldots, n$.
- Define the **Riemann sum** by $\sum_{i=1}^{n} f(x_i^*)\Delta x$
- Define the integral of f by

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \int_a^b f(x)\, dx.$$

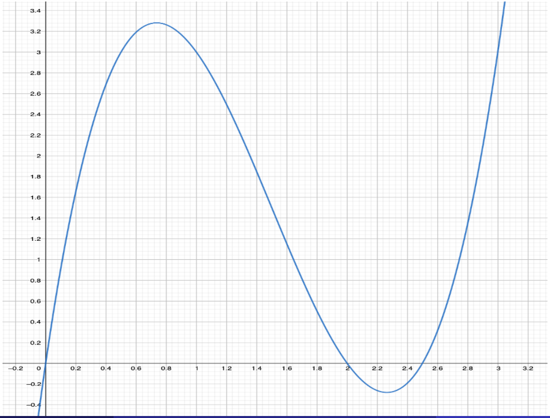
- We interpret the integral as the area "under" the graph of f.

Net Change Theorem

$$\int_a^b f'(x)\,dx = f(b) - f(a).$$

The integral of a rate of change gives the net change.

Example: The graph below gives the rate of growth of a population in years. Approximate the net growth in the first three years.

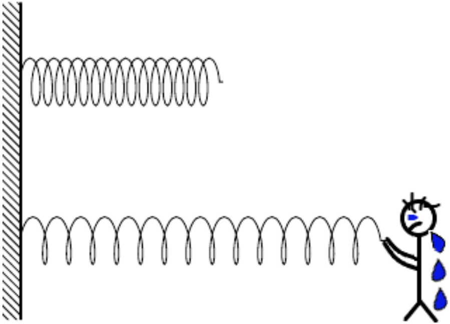


Average value of a function

The average value of a function f over $[a, b]$ is

$$\text{Ave} = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

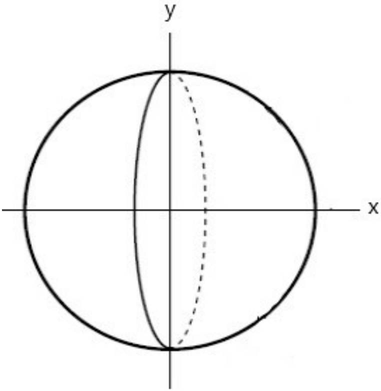
Example: Find average work done to stretch a spring 5 meters from its rest position, if the force required at distance d from rest is given by $F = kx$ for some constant k. (Work = Force x distance)



Volume by slicing

Idea: Approximate a volume by cutting it into thin slices and adding up the cross-sectional areas of all the slices.

Examples: (i) Approximate the volume of a sphere with radius $r = 1$. Express the volume as an integral.



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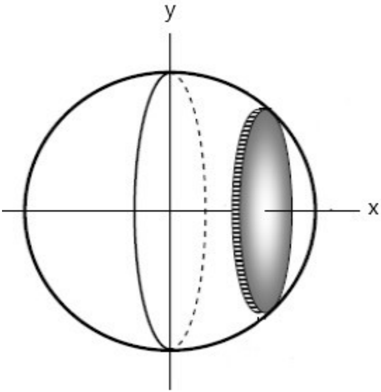

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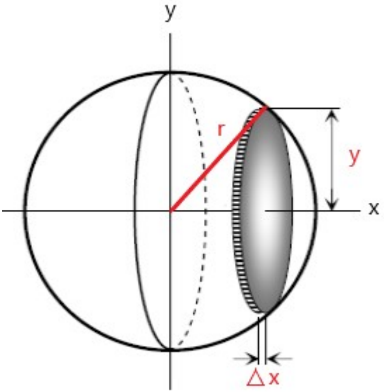

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Examples: (ii) Approximate the volume of the solid formed by rotating $y = \sqrt{x}$ about the x-axis with $2 \leq x \leq 10$. Express the volume as an integral