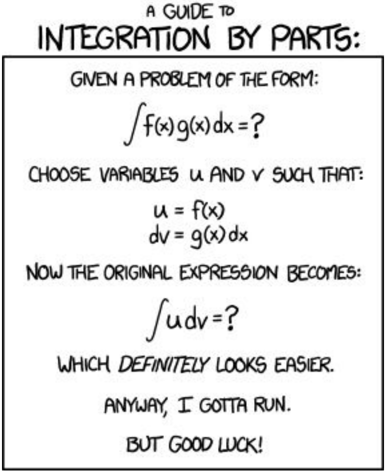
Integration by Parts



Question:

$$\int xe^x \, dx = ?$$

Integration by parts

$$\int u \, dv = uv - \int v \, du$$

Let

$$\begin{aligned} u &= x & \underline{dv} &= e^x dx, \text{ then} \\ \underline{du} &= dx & v &= e^x \end{aligned}$$

Then

$$\int \underset{u}{x} \, \underset{\underline{dv}}{e^x \, dx} = \underset{u}{x}\underset{v}{e^x} - \int \underset{v}{e^x} \, \underset{\underline{du}}{dx} = xe^x - e^x + C$$

Integration by parts

$$\int u\, dv = uv - \int v\, du$$

Let $u = x$, then $du = dx$ and
let $dv = e^x dx$, then (by "usual" integration) $v = e^x$. So

$$\int xe^x\, dx = xe^x - \int e^x dx = xe^x - e^x + C$$

For a definite integral:

$$\int_0^1 xe^x\, dx = [xe^x]_0^1 - \int_0^1 e^x dx = [xe^x - e^x]_0^1 = 1$$

Example

$$\int x \sin(2x) dx = ?$$

$$\begin{aligned} u &= x & dv &= \sin(2x)dx \\ du &= dx & v &= -\frac{1}{2}\cos(2x) \end{aligned}$$

Then

$$\begin{aligned} \int x\sin(2x)dx &= -\frac{1}{2}x\cos(2x) + \frac{1}{2}\int \cos(2x)\, dx \\ &= -\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) + C \end{aligned}$$

Example

$$\int ln(x)dx = ?$$

$$\begin{aligned} u &= ln(x) & dv &= 1\,dx \\ du &= \frac{1}{x}dx & v &= x \end{aligned}$$

Then

$$\begin{aligned} \int ln(x)dx &= xln(x) - \int x\left(\frac{1}{x}\right)\,dx \\ &= xln(x) - x + C \end{aligned}$$

Example

$$\int \arctan(x)dx = ?$$

$$\begin{aligned} u &= \arctan(x) & dv &= 1\,dx \\ du &= \frac{1}{1+x^2}\,dx & v &= x \end{aligned}$$

Then

$$\begin{aligned} \int ln(x)dx &= x\arctan(x) - \int \frac{x}{1+x^2}\,dx \\ &= x\arctan(x) - \frac{1}{2}ln(1+x^2) + C \end{aligned}$$

Example

$$\int e^x \cos(x)\, dx = ?$$

$$\begin{aligned} u &= e^x & dv &= \cos(x)\, dx \\ du &= e^x dx & v &= \sin(x) \end{aligned}$$

Then

$$\begin{aligned} \int e^x \cos(x)\, dx &= e^x \sin(x) - \int e^x \sin(x)\, dx \\ &= e^x \sin(x) - \left[-e^x \cos(x) + \int e^x \cos(x)\, dx \right] \end{aligned}$$

Thus

$$\int e^x \cos(x)\, dx = \frac{1}{2} e^x \sin(x) + \frac{1}{2} e^x \cos(x)$$

Exercises

1. $\int x^4 ln(x)\,dx = ?$
2. $\int x^2 e^{3x}\,dx = ?$
3. Prove the reduction formula $\int x^n e^x\,dx = x^n e^x - n\int x^{n-1}e^x\,dx$
4. $\int x^6 e^x\,dx$ (use the reduction formula from the previous problem)