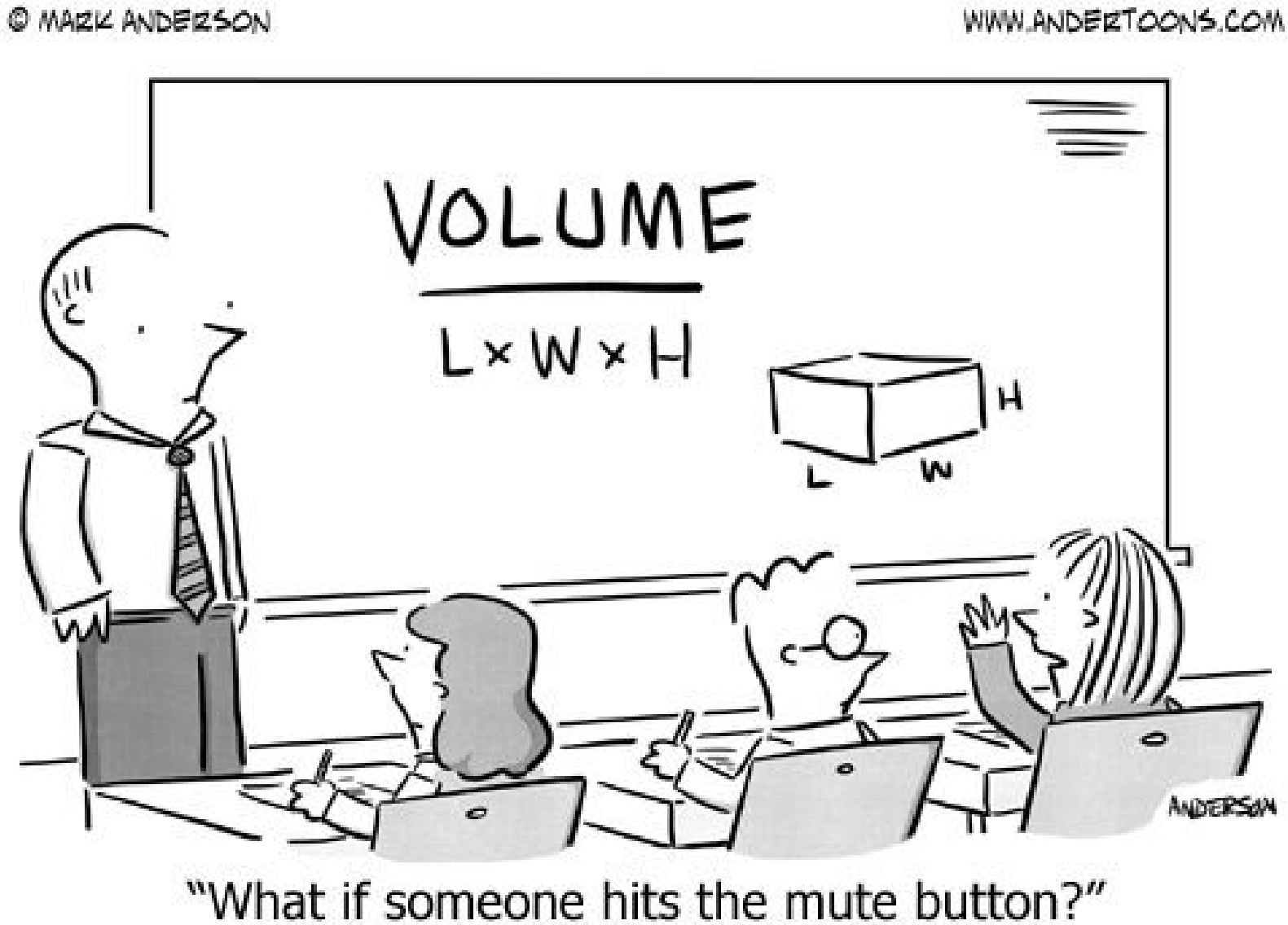
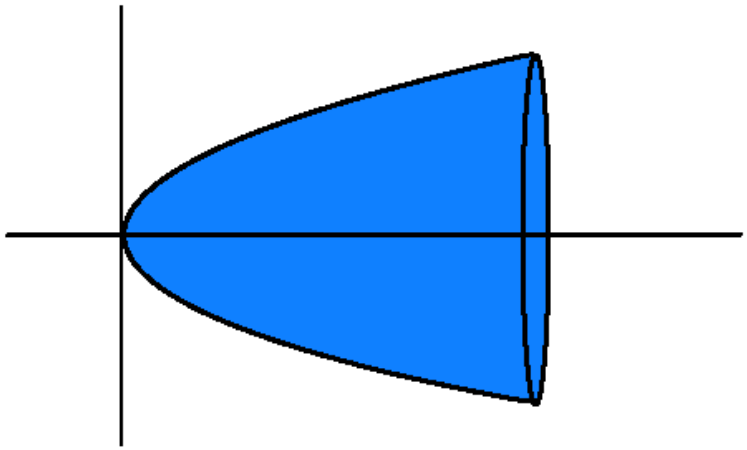
Volumes of Revolution



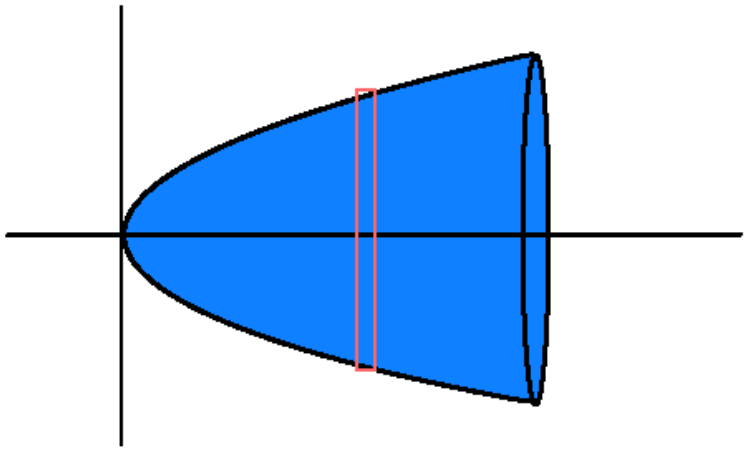
Revolution about the x-axis

Consider the solid formed by revolving the region bounded by $0 \leq x \leq 1$, the x-axis and $y = \sqrt{x}$ about the x-axis.



Volume by disks - revolved about the x-axis

Consider the solid formed by revolving the region bounded by $0 \leq x \leq 1$, the x-axis and $y = \sqrt{x}$ about the x-axis .

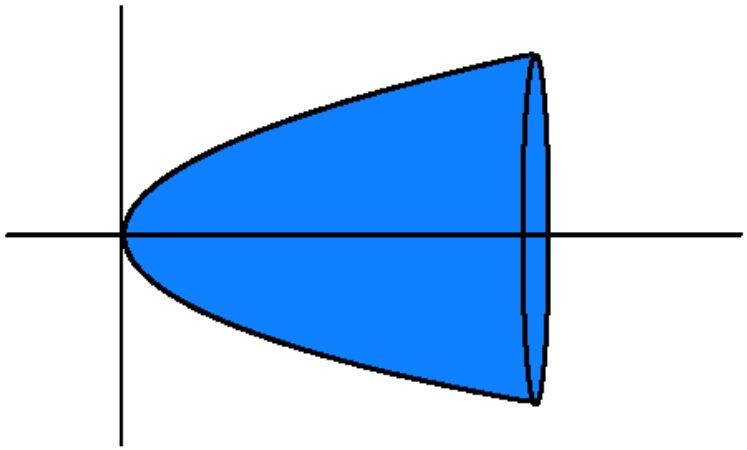


Cross-sectional area of one disk: $A(x) = \pi y^2 = \pi(\sqrt{x})^2$
Volume of one disk: $V = A(x)\Delta x$ (i.e. area $\times$ thickness)
Volume of the whole solid:

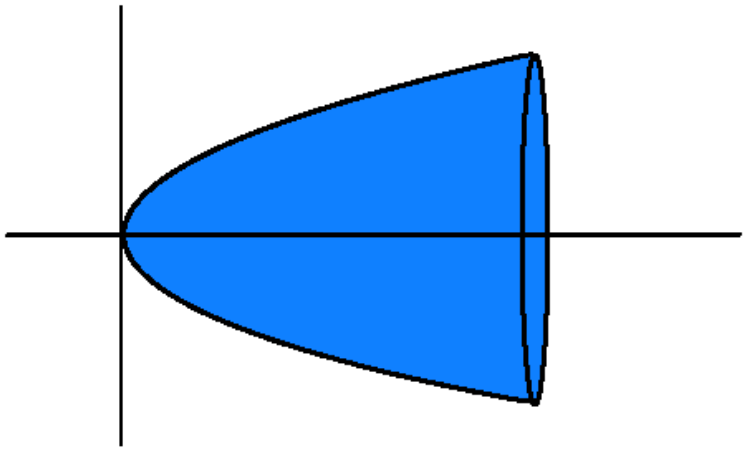
$$V = \int_0^1 A(x)\, dx = \int_0^1 \pi x\, dx = \frac{\pi}{2}$$

Volume by cylindrical shells - revolved about the x-axis

Consider the solid formed by revolving the region bounded by $0 \leq x \leq 1$, the x-axis and $y = \sqrt{x}$ about the x-axis .

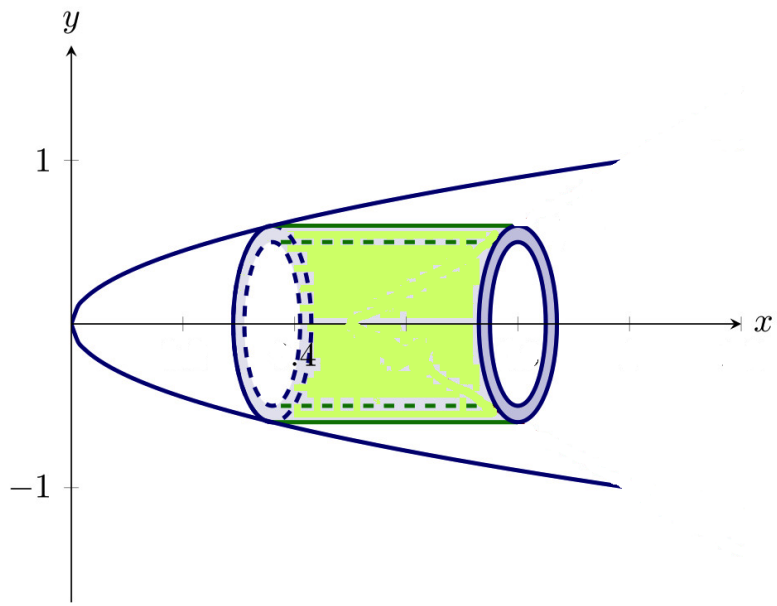


Idea:



Volume by cylindrical shells - revolved about the x-axis

Consider the solid formed by revolving the region bounded by $0 \leq x \leq 1$, the x-axis and $y = \sqrt{x}$ about the x-axis.

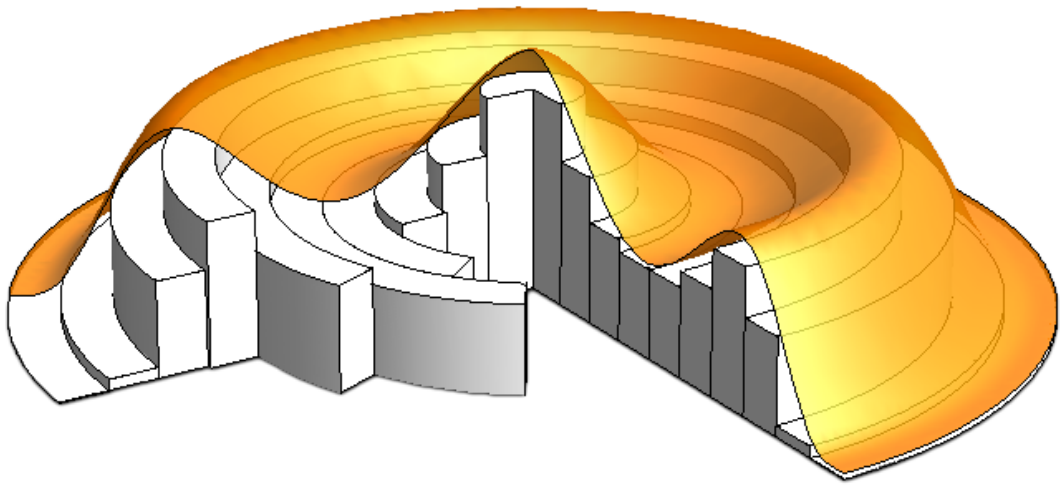


Volume of one cylindrical shell: $V = 2\pi y^*(1 - (y^*)^2)\Delta y$ (i.e. circumference $\times$ length$\times$ thickness)
Volume of the solid:

$$V = \int_0^1 2\pi(y - y^3)\, dy = \frac{\pi}{2}$$

Volume by cylindrical shells - revolved about the y-axis

Consider the solid formed by revolving the region bounded by $0 \leq x \leq 2$, the $y = 0$ and $y = 2x^2 - x^3$vabout the y-axis. Idea:



Volume by cylindrical shells - revolved about the y-axis

Consider the solid formed by revolving the region bounded by $0 \leq x \leq 2$, the $y = 0$ and $y = 2x^2 - x^3$.

Volume of one cylindrical shell:

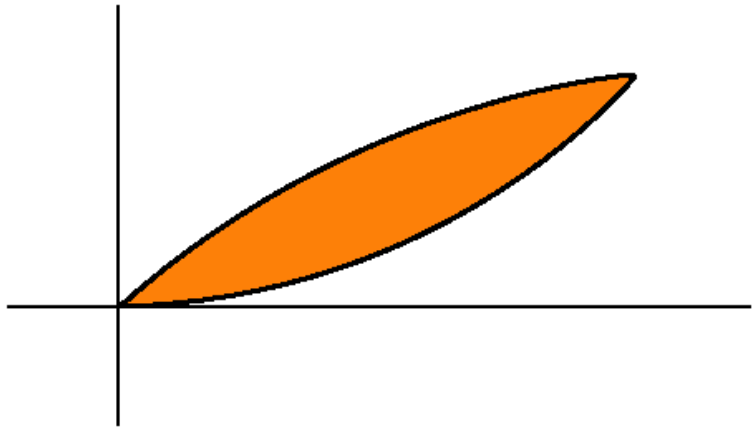
$$V = 2\pi \; x \; y(x) \; \Delta x = 2\pi x(2x^2 - x^3)\Delta x$$

(i.e. circumference $\times$ length $\times$ thickness)
Volume of the solid:

$$V = \int_0^2 2\pi x(2x^2 - x^3) \, dx = \frac{16\pi}{5}$$

Exercises

Consider the region R bounded by $0 \leq x \leq 1$ and $x^2 \leq y \leq \sqrt{x}$.



1. Find the volume of the solid obtained by revolving R about the x-axis.
2. Find the volume of the solid obtained by revolving R about the y-axis.