Math 8 - Review

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7 questions to students:

1. What is a derivative?

For a continuous (and differentiable) function, the derivative $f'(a)$ is the instantaneous rate of change of $y=f(x)$ with respect to x when $x=a$. It can be computed using the formula

Last question of the quiz

$$f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}.$$

2. What two objects do the Fundamental Theorem of Calculus link?

Theorem

If f is continuous on $[a,b]$, then the function g defined by

$$g(x) = \int_a^x f(t)\,dt, \quad a \le x \le b$$

is continuous and differentiable on (a,b), with $g'(x)=f(x)$.

3. What is the limit of a function?

The limit of $f(x)$, as x approaches a, is L if, when x goes arbitrary close to a (without necessarily reaching a), $f(x)$ gets arbitrary close to L, no matter the side it comes from.

Alternative / formal definition

L is the limit $L=\lim_{x \to a} f(x)$ if, for any $\varepsilon > 0$, there exists $\delta > 0$ such that, if $|x-a| < \delta$, then

$$|f(x)-L| < \varepsilon.$$

4. What is the derivative of $f(g(x))$?

Chain rule

If f and g are differentiable in x,

$$(f(g(x)))' = f'(g(x)) \cdot g'(x).$$

Proof

Assume $g(x)$ is not constant (if it is, then the derivative is simply 0).

Then,

$$\frac{(f(g(x)))'}{g'(x)} = \lim_{x \to a} \frac{f(g(x)) - f(g(a))}{g(x) - g(a)} = f'(g(x)).$$

Therefore,

$$(f(g(x)))' = f'(g(x))\, g'(x)$$

Example

$$(\sin(e^{2x}))' = \sin'(e^{2x}) \cdot (e^{2x})'$$

$$= \cos(e^{2x}) \cdot e^{2x} (2x)'$$

$$= 2e^{x} \cos(e^{2x})$$

Example

$$e^{\ln(x)} = e^{\ln(x)} (\ln(x))'$$

$$= \frac{e^{\ln(x)}}{x}$$

$$= 1 \qquad (\text{since } e^{\ln(x)} = x)$$

But it would be easier to change $e^{\ln(x)}$ to x first, and then get $x' = 1$.

5. Summation notation

$$\sum_{i=a}^{b} f(i) = f(a) + f(a+1) + f(a+2) + \dots + f(b-1) + f(b)$$

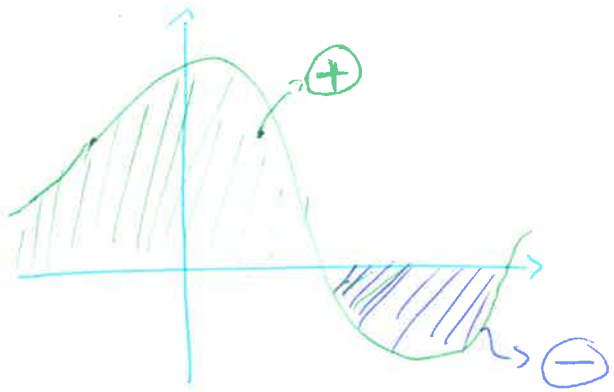
of course, $a < b$ here.

Example:

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \dots + (n-1)^2 + n^2$$

6. Integral

An informal (but still honest...) way of defining an integral is as the signed area between the curve and the x-axis.



7. Fundamental theorem of calculus (continued)

By the FTC, the derivative of $\int f(x)\,dx$ is just $f(x)$