Math 8. Lecture 10

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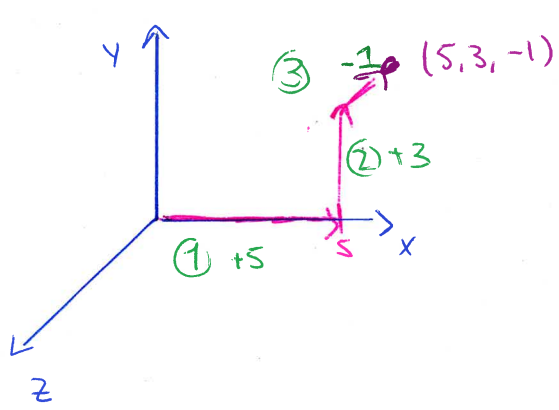
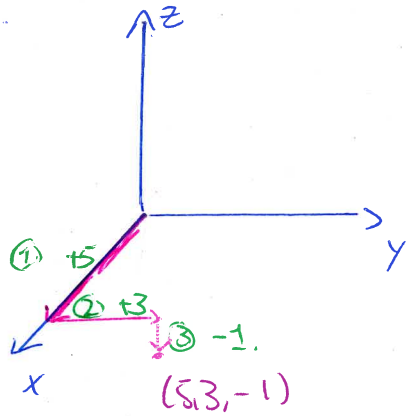
3D surfaces

I- Coordinates





Where is (5,3,-1)?

Are they the same? We can rotate the projection of the space on the paper of 120°.

To draw a point:

The point (a,b,c) is drawn by moving from the origin:

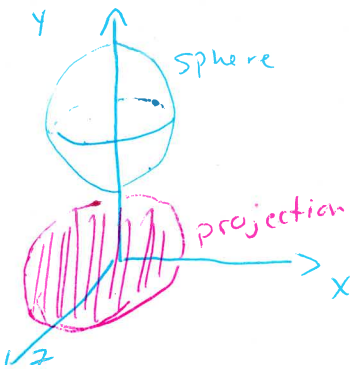
① one step of size a along the x-axis,
② b in the same direction as the y-axis, and
③ c in the direction of the z-axis.

II - Projection.

The projection of a geometric object (point, line, surface, ...) onto a plane is a flattening of the object in the direction perpendicular to the plane.

Examples

· A sphere onto the plane xz



For a point:

- The point (a,b,c), when projected on the yz-plane, is (0,b,c).
- What does the line (in dimension 3) x=2, y=3 looks like on the plane perpendicular to the z-axis?
This plane is xy, and so the projection of $\{(2,3,z) \mid z \in \mathbb{R}\}$ is the point (2,3).

III- Surfaces.

An equation in x, y and z represents a surface in $\mathbb{R}^3$.

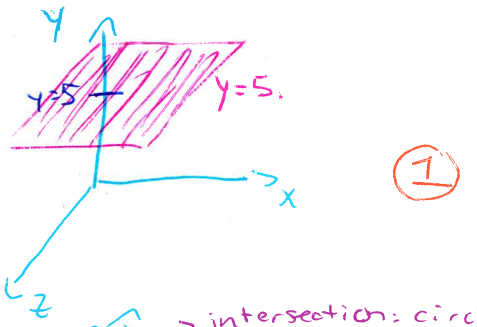
Example of surfaces

- plane
- sphere
- cylinder
- hosse saddle / pringle
- ...

Examples

1- what is $y=5$ in $\mathbb{R}^3$?

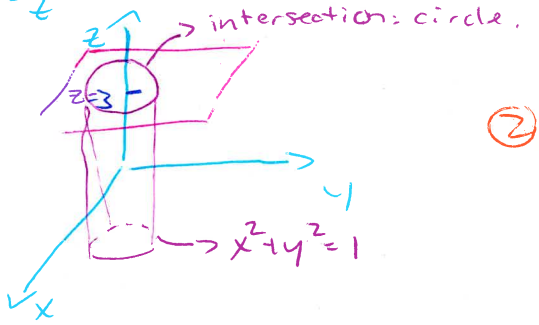
A plane.



(1)

2- What is the intersection of the surfaces $x^2+y^2=1$ and $z=3$?

A circle.



(2) (3)

3- What is $x^2+y^2=1$?

A cylinder, of radius 1.

4- What is $y=x$?

A plane

IV - Distance in three dimensions

The distance $|P_1P_2|$ between two points $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}.$$

Example:

The distance between $P(2,-1,7)$ and $Q(1,-3,5)$ is

$$|PQ| = \sqrt{(2-1)^2 + (-1-(-3))^2 + (7-5)^2}$$
$$= \sqrt{1+4+4}$$
$$= 3.$$

V- Spheres.

A sphere is the set of all points at a given distance r, called the radius, from a point (h,k,l), that is its center.

Its equation (as a surface in 3D) is

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$$

If the center is the origin ($O(0,0,0)$), the equation of a sphere of radius r is

$$x^2 + y^2 + z^2 = r^2$$

Example:

Where is centered $x^2+y^2+z^2+4x-6y-3=0$? What is its radius?

$$x^2+y^2+z^2+4x-6y-3 = x^2+4x+4 + y^2-6y+9 + z^2 - 16 \quad \text{(completing the squares)}$$
$$\Rightarrow (x+2)^2 + (y-3)^2 + z^2 = 4^2.$$

This is the equation of a sphere of radius 4 centered at $(-2,3,0)$

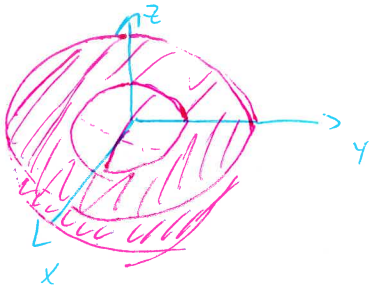
Example

What region of $\mathbb{R}^3$ is represented by

$$1 \leq x^2+y^2+z^2 \leq 4?$$

What is its volume?

Solution: This is the region inside the sphere of radius 2, but outside the sphere of radius 1



Its volume is the difference of volume of the two spheres:

$$\frac{4\pi(2)^3}{3} - \frac{4\pi(1)^3}{3} = \frac{28\pi}{3}.$$

Reference: James STEWART. Calculus, 8th edition.
Section 12.1