Math 8
Winter 2020
Section 1
January 27, 2020

First, some important points from the last class:

An application of the method of taking limits of Riemann sums to find integrals is to computing work of various kinds.

The work done by a constant force of magnitude F (acting in the direction of motion) on an object that moves a distance d is given by

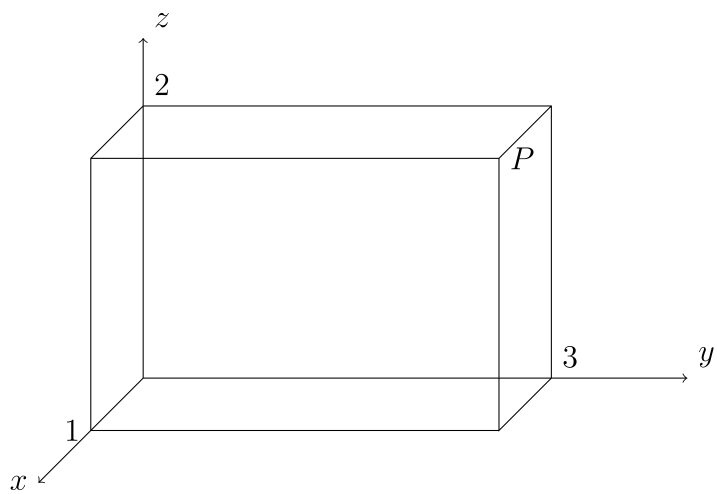
$$W = Fd.$$

If the force on the object is not constant, or if not all parts of the object move the same distance, then we can use Riemann sums.

A second application of integration we saw last class is probability density functions. If $P(x)$ is a probability density function for some random number, then the total area under the graph of P is 1, and the probability the number is between a and b is the integral

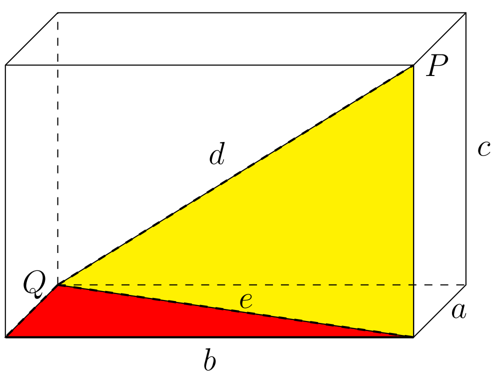
$$\int_a^b P(x)\, dx.$$

Three-dimensional coordinate system (x-axis points out of paper):



$$P = (1, 3, 2)$$

Distance between points:



$$d^2 = e^2 + c^2 = (a^2 + b^2) + c^2 \qquad d = \sqrt{a^2 + b^2 + c^2}$$

Distance from $Q = (x_1,\, y_1,\, z_1)$ to $P = (x_2,\, y_2,\, z_2)$ is

$$\boxed{|QP| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

$\mathbb{N} = \{0, 1, 2 \ldots\}$ is the set of natural numbers (N is for number).
$\mathbb{Z} = \{\cdots -2, -1, 0, 1, 2 \ldots\}$ is the set of all integers (Z is for *Zahlen*, number in German).
$\mathbb{Q}$ is the set of rational numbers, or fractions (Q is for quotient).
$\mathbb{R}$ is the set of all real numbers (yes, R is for real).
$\mathbb{R}^3$ denotes 3-dimensional space, or the set of all triples (a, b, c) of real numbers.

Equations and inequalities in $\mathbb{R}^3$:

$z = 0$

$z = 3$

$x = 1$

$x = y$

$0 \leq y < 1$

$x^2 + y^2 = 4$

$4 \leq x^2 + y^2 \leq 9$

$y = x^2$

$x^2 + y^2 + z^2 = 4$

$x^2 + y^2 + z^2 \leq 4$

$(x - 1)^2 + (y - 2)^2 + (z + 1)^2 = 1$

Sphere of radius r with center (a, b, c):
Distance from (x, y, z) to (a, b, c) is r.

$$\sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} = r$$

$$\boxed{(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2}$$

Example: Suppose S is a sphere, the center of S is $(1, -1, 2)$, and the point $(2, 3, 0)$ lies on S. Find an equation for S.

Example: Find the center and radius of the sphere with equation

$$x^2 + 2x + y^2 + z^2 - 6z = 6$$

Visualizing surfaces in $\mathbb{R}^3$:

Try drawing the surface. It may help to sketch the intersection of the surface with the coordinate planes. It may also help to sketch horizontal cross-sections, or cross-sections parallel to the xz- or yz-planes.

Example:

$$z^2 = x^2 + y^2 + 1$$

First sketch the intersection with the yz-plane $x = 0$. Then sketch horizontal cross-sections.

A horizontal cross-section at height k is in the plane $z = k$. It satisfies the two equations

$$z = k \qquad k = x^2 + y^2.$$

Think of the horizontal plane $z = k$ as a copy of the xy-plane and sketch the curve $k = x^2 + y^2$.

Example:

$$x^2 + z^2 = y$$

First sketch the intersection with the yz-plane. Then sketch cross-sections in vertical planes $y = k$ (parallel to the xz-plane).

Exercise: Sketch and describe the surface $x^2 + y^2 - z^2 = 0$

Exercise: The surface $x^2 + y^2 - z^2 = 0$ from the last exercise intersects the plane $z = 3$ in a circle. Give the equation of a sphere containing that circle. (This means the circle lies on the surface of the sphere.) There are many possible answers.

Now give the equation of a different sphere containing that same circle. (Think geometrically.)

Exercise: Sketch the surface

$$z^2 = x^2 + y^2 - 1$$

Exercise: What geometric object is described by $x = y$?

By $y = 2z + 1$?

By $x = y = 2z + 1$?

Find a point satisfying $x = y = 2z + 1$ whose distance from the origin is $\sqrt{3}$. (How many such points are there?)

Exercise: What geometric object is described by

$$\{(t, -t, 2t) \mid t \in \mathbb{R}\}?$$

(This is read as "the set of all $(t, -t, 2t)$ such that t is in $\mathbb{R}$." It is the set of all points of the form $x = t, y = -t, z = 2t$, for any real number t.)