Practice exam for unit 2

These are problems like those that might occur on an exam. This collection of problems is not intended to have the same length, or cover precisely the same topics, as the actual exam.

1 True or false questions

Indicate whether the following statements are TRUE (T) or FALSE/NOT NECESSARILY TRUE (X). You do not need to justify your answer.

(a) The line with vector equation $\mathbf{r}(t) = \langle 1-2t, -1+6t, 2+4t \rangle$ is orthogonal to the plane $x - 3y - 2z = 1$.

(b) A projectile is fired from a cannon in an arc with velocity given by a vector function $\mathbf{v}(t)$, as a function of time. The distance along the ground that the projectile travels as a function of time can be obtained by integrating its speed.

(c) The vector functions $\mathbf{r}_1(t) = \langle t, te^{2t} \rangle$ and $\mathbf{r}_2(s) = \langle s-1, se^{2s-2} - e^{2s-2} \rangle$ describe the same curve in $\mathbb{R}^2$.

(d) The magnitude of the projection of $\vec{v}$ onto $\vec{w}$ always equals the component of $\vec{v}$ in the direction of $\vec{w}$.

(e) Suppose a particle moves in 3-dimensional space with position vector $\mathbf{r}(t)$. If the particle's velocity $\mathbf{v}(t)$ is parallel to its acceleration $\mathbf{a}(t)$, then it moves in a straight line.

(f) $x^2 + y^2 + 4z^2 = 0$ is the equation of a sphere.

(g) Given a curve with position vector $\mathbf{r}(t)$ parameterized over t, if the derivative of its unit tangent vector is zero everywhere, then the curve is a straight line.

(h) A particle moves in space with position vector

$$\mathbf{r}(t) = \langle \cos(2t), \sin(2t), \cos(2t) \rangle,$$

parameterized by time $t \geq 0$. At any point in time the particle's acceleration is in the opposite direction to its position vector.

2 Multiple choice questions

Choose the best answer.

1. If two lines in $\mathbb{R}^3$ are not parallel, then

   (a) They intersect at a single point.

   (b) They do not intersect.

   (c) Either of the above could be true.

2. The following line is perpendicular to the plane $x - 4y + 3z = 4$.

   (a) $\vec{r} = \langle 1, -4, 3\rangle + t\langle 1, 1, -1\rangle$.

   (b) $\vec{r} = \langle -2, 3, 0\rangle + t\langle 1, 1, 1\rangle$.

   (c) Neither of the above.

3. If ℓ is a line in $\mathbb{R}^3$ in the direction of the vector $\vec{v}$, and $\mathcal{P}$ is a plane in $\mathbb{R}^3$ with normal vector $\vec{n}$, and ℓ intersects $\mathcal{P}$ at a single point, then

   (a) $\vec{v}$ and $\vec{n}$ must be parallel.

   (b) $\vec{v}$ and $\vec{n}$ must be perpendicular.

   (c) $\vec{v}$ and $\vec{n}$ may or may not be parallel, but cannot be perpendicular.

   (d) $\vec{v}$ and $\vec{n}$ may or may not be perpendicular, but cannot be parallel.

3 Written problems

1. A moving particle has position function $\vec{r}(t) = \langle t\cos t, t\sin t, t\rangle$.

   (a) Find the object's velocity and acceleration functions $\vec{v}(t)$ and $\vec{a}(t)$.

   (b) Find the object's speed at time t.

   (c) Write down an integral that gives the arc length of the object's path between the points $(0, 0, 0)$ and $(-\pi, 0, \pi)$. Do not evaluate this integral.

   (d) At time $t = 0$, find the object's velocity and acceleration, and the unit tangent vector to the object's path $\vec{T}$.

   (e) Find the tangential and normal components of the object's acceleration when it is at the point $(0, 0, 0)$.

(f) Find the curvature of the object's path at the point $(0, 0, 0)$.

2. Find a function $\vec{r}(t)$ parametrizing the curve formed by the intersection of the surfaces with equations $z = \sqrt{x^2 + y^2}$ and $z^2 = 2x + 2y + 2$.

3. Find the work done by the force of gravity $\langle 0, 0, -mg\rangle$ on an object of mass m sliding down a straight ramp from the point $(0, 0, 1)$ to the point $(2, 3, 0)$.

4. Find a linear equation of the plane containing the points $A = (1, 0, -2), B = (1, 3, 1)$ and $C = (-1, 3, -2)$.

5. Find a function parametrizing:

   (a) the curve formed by the intersection of the sphere $x^2 + y^2 + z^2 = 10$ and the plane $z = 1$.

   (b) the curve formed by the intersection of the planes $2x+y-z = 3$ and $x-2y+z = 0$.

6. Let $\vec{u}$ and $\vec{v}$ be unit vectors, and $\vec{u} \cdot \vec{v} = \frac{1}{2}$. What is $|\vec{u} \times \vec{v}|$?

7. Find the angle B of the triangle whose vertices are $A = (7, 0, 6)$, $B = (5, 0, 4)$ and $C = (4, 1, 2)$.

8. At what points does the helix $r(\vec{t}) = \langle \cos(t), \sin(t), t\rangle$ intersect the surface $x^2+y^2+z^2 = 10$?

9. (a) Sketch the curve $r(\vec{t}) = \langle \sqrt{t}, \sin(t)\rangle$.

   (b) Find a tangent line to the curve at point $(\sqrt{2\pi}, 0)$.

   (c) Give a position function of a particle travelling on that curve from $(\sqrt{2\pi}, 0)$ to $(0, 0)$. Don't forget to specify the domain.

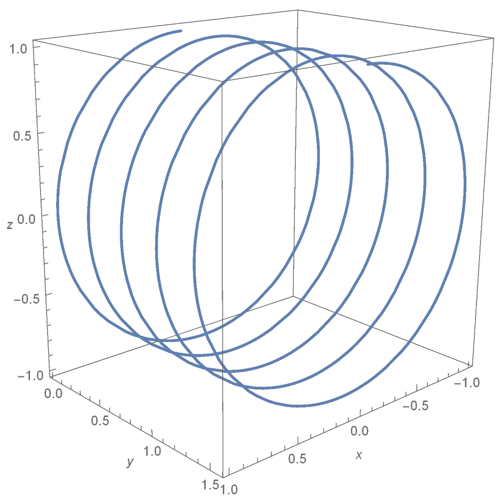
10. An artist needs for a project a long ribbon that will be arranged in the shape of the curve $\langle t^2, \frac{2t^3}{3}, \frac{t^4}{4}\rangle$, with the z-axis pointing up. The total creation should be 4 meters high, and the ribbon will cover the total height of the creation, from the top of the creation to the floor. How long should the ribbon be?

11. (a) Give a function parametrizing the curve $\frac{(x-2)^2}{16} + \frac{y^2}{9} = 4$.

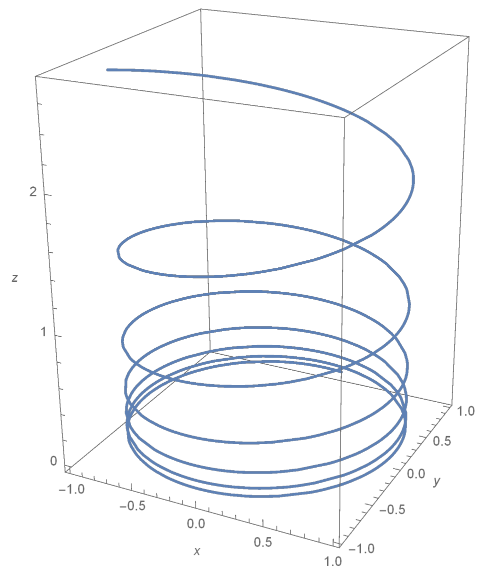
(b) Describe this curve in words. Be as specific as possible.

4 Picture matching problem

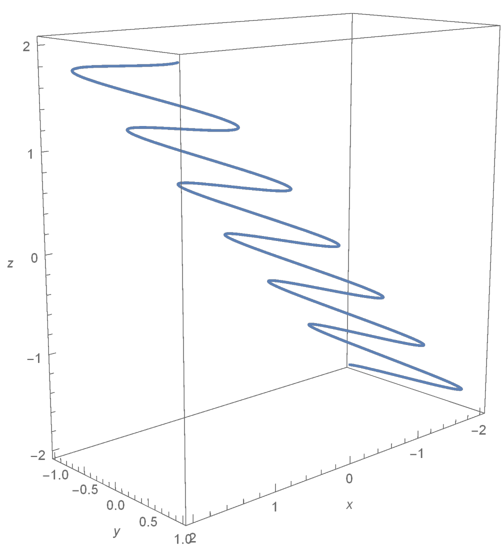
Match the following pictures with their description. Note that there are more options to choose from than pictures.



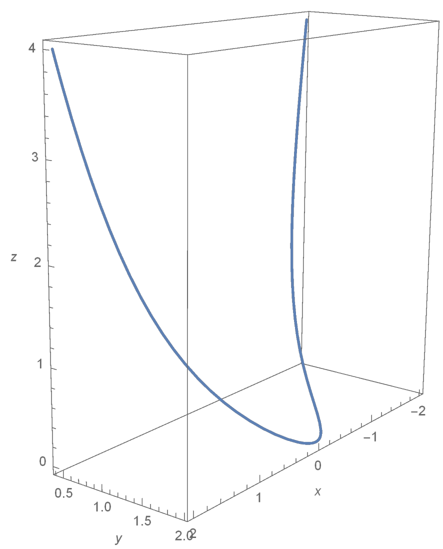
a)



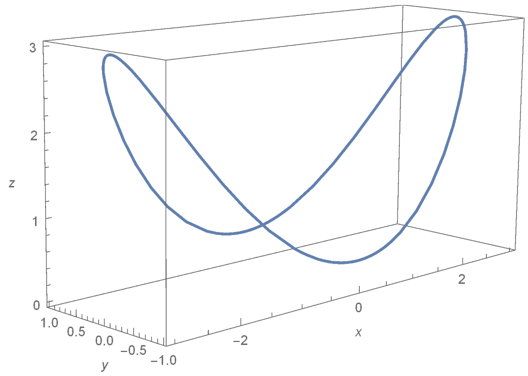
b)



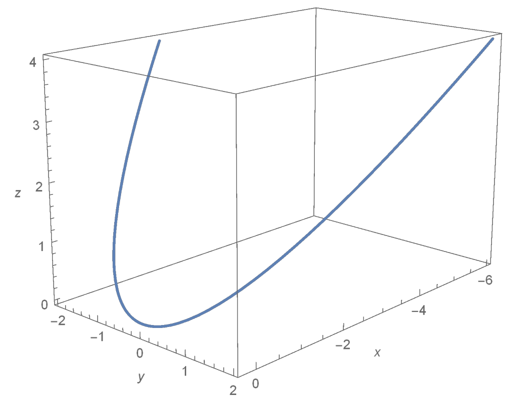
c)



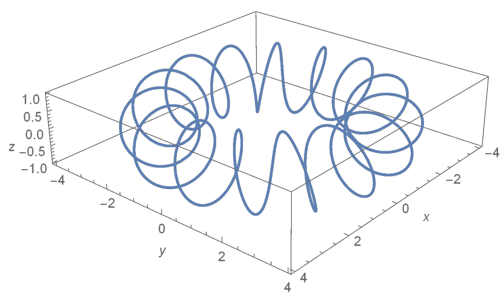
d)



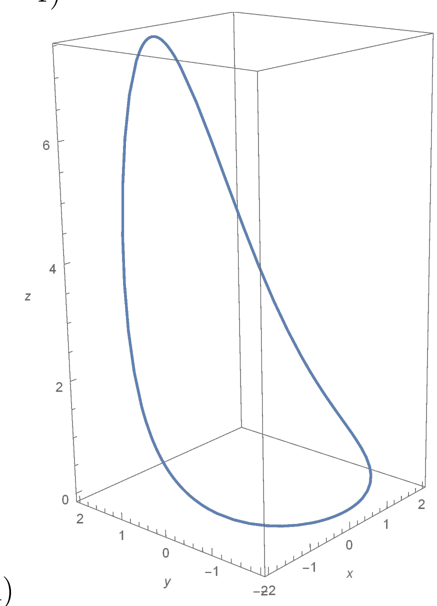
e)



f)



g)



h)

1. $z^2 = 1 + x^2 + y^2$
2. The intersection of $y^2 = z$ and the plane containing the origin and normal to the vector $\vec{v} = \langle 1, 1, 1 \rangle$
3. $\frac{x^2}{4} + y^2 + z = 0$
4. $\frac{x^2}{9} + \frac{(y-2)^2}{4} + (z-3)^2 = 1$
5. The intersection of $x = z$ and $y = \sin(x)$
6. The cylinder whose trace in the x-y plane is $\frac{x^2}{4} + y^2 = 1$
7. $\vec{r}(t) = \langle \cos(t), \sin(t), e^t \rangle$
8. The intersection of $x^2 + y^2 = 2$ and $z = e^y$
9. $\frac{x^2}{4} + y^2 - z^2 = 1$

10. The intersection of the the paraboloid $3z = x^2 + y^2$ and the cylinder $\frac{x^2}{9} + y^2 = 1$
11. The sphere with radius 3 centered at $(0, 2, 3)$
12. $x = \sin(2z)$
13. $\vec{r}(t) = \langle (3 + \sin(15t)) \cos(t), (3 + \sin(15t)) \sin(t), \cos(15t) \rangle$
14. The plane containing the origin with a normal vector $\langle 1, -3, 1 \rangle$
15. $\vec{r}(t) = \langle \sin(t), t, \cos(t) \rangle$
16. $\vec{r}(t) = \langle t, \frac{2}{1+t^2}, t^2 \rangle$