

PROJECT 2
THE COCKROACH ON AN ELASTIC TIGHTROPE

MATH 9 FALL 2001

Directions: You may either work alone on this project or with one other person (i.e. groups of no more than 2 people). For this project, you are allowed to use any materials you want, books, notes, etc. and you are allowed to discuss the project with the other member of your group and with Professor Pauls. For this project, you are *not* allowed to discuss the project with the tutors at the tutorial sessions. You must give an explanation of the answers to the questions which reflect *your own understanding* of the answer. In other words, if asked, you should be able to reproduce and explain the answers that you turn in. As with the first project, you should explain your procedure and computations as completely as possible. Any violation of these directions will be considered a violation of the Honor Principle.

The project is due Monday November 19 at the beginning of class.

There are two parts to the project, part B is a more general version of part A. Before beginning either part, answer the following question for yourself. Once you understand the answer, you are ready to try the main parts of the problem.

Question: *Suppose you are standing on an elastic tightrope b meters from the left end and c meters from the right end. At that moment, the tightrope instantly stretches uniformly, increasing its length by d meters. How far are you now from each end?*

Part A:

Suppose a cockroach starts at one end of a 1000 meter tightrope and runs towards the other end at a speed of one meter per second. At the end of every second, the tightrope stretches uniformly and instantaneously, increasing its length by 1000 meters each time.

- (1) Does the roach ever reach the other end?
- (2) If so, about how long does it take?

To answer these questions, proceed as follows. Consider the sequence $\{d_i\}_{i=1}^{\infty}$ where d_i represents the distance the roach still has to go after i seconds, but before the rope does its instantaneous stretch. Find a formula for $\{d_i\}$ and then simplify it.

For absolutely full credit, *prove* your formula is correct by using mathematical induction (read about induction in the handout available in class).

Part B:

Suppose you don't know how long the rope is initially, nor how fast the roach runs, nor how much longer the rope gets after every second. All you know is that the roach maintains a constant speed, and that the rope stretches uniformly and instantaneously by some fixed amount after every second. Answer the same two questions from Part A.