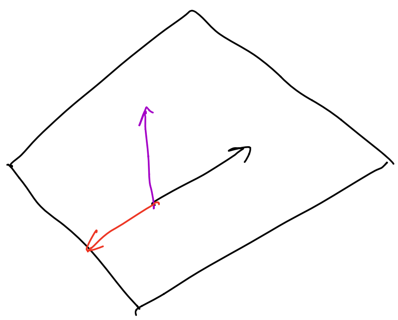
1. Linear algebra 2.1.9 (c) (d) Solution

(c) Note that there exist a pair of non-parallel vectors $\langle 3,1,2\rangle$, $\langle -1,2,1\rangle$. Since the third vector $\langle 2,-4,-2\rangle = -2\langle -1,2,1\rangle$, three vectors are coplanar.



← coplanar

(d) Again, there exist a pair of non-parallel vectors, $\langle 3,1,2\rangle$, $\langle -1,2,1\rangle$. Now, we check if the third vector is a linear combination of the first two.

Solve $\langle 1,4,4\rangle = s\langle -1,2,1\rangle + t\langle 3,1,2\rangle$

$$\left[\begin{array}{cc:c} -1 & 3 & 1 \\ 2 & 1 & 4 \\ 1 & 2 & 4 \end{array}\right] \xrightarrow[R_3 \to R_3 + R_1]{R_2 \to R_2 + 2R_1} \left[\begin{array}{cc:c} -1 & 3 & 1 \\ 0 & 7 & 6 \\ 0 & 5 & 5 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3/5]{} \left[\begin{array}{cc:c} -1 & 3 & 1 \\ 0 & 7 & 6 \\ 0 & 1 & 1 \end{array}\right] \xrightarrow[R_2 \leftrightarrow R_3]{} \left[\begin{array}{cc:c} -1 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 7 & 6 \end{array}\right] \xrightarrow{R_3 \to R_3 - 7R_2} \left[\begin{array}{cc:c} -1 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{array}\right]$$

The system is inconsistent, therefore, the set is not coplanar.

2. Linear Algebra 2.1.10 (d)(e)

(d) First, there exist a pair of non-parallel vectors. $\langle 1,1,1\rangle$ and $\langle 1,-1,0\rangle$.

Furthermore, $\langle 2,0,1\rangle = \langle 1,-1,0\rangle + \langle 1,1,1\rangle$.

Hence A is coplanar, and span (A) is a plane through the origin.

(e) Pick a pair of non-parallel vectors $\langle 1,1,1\rangle$ and $\langle 2,0,1\rangle$. Since other vectors are scalar multiple of them, the set is coplanar.

Hence Span (A) is a plane through the origin.

3. Note that

$$\begin{aligned} \langle 3,2,5\rangle &= \langle 1,1,2\rangle + \langle 2,1,3\rangle \\ \langle -1,0,-1\rangle &= \langle 1,1,2\rangle - \langle 2,1,3\rangle. \end{aligned} \qquad (*)$$

Therefore, $\langle 1,1,2\rangle$ and $\langle 2,1,3\rangle$ are linear combinations of $\langle 3,2,5\rangle$ and $\langle -1,0,-1\rangle$.

Let $A = \{\langle 3,2,5\rangle, \langle -1,0,-1\rangle\}$ and $B = \{\langle 1,1,2\rangle, \langle 2,1,3\rangle\}$.

Now suppose v is a linear combination of A. Then $(*)$ implies that v is also a linear combination of B.

In particular, $\boxed{\text{span }(A) \subseteq \text{Span }(B)}$

subset notation

$(X \subseteq Y$ if for every $x \in X$, x is also in $Y)$

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Look at (*) again,

$$\langle 1,1,2\rangle = \frac{1}{2}\left(\langle 3,2,5\rangle + \langle -1,0,-1\rangle\right)$$
$$\langle 2,1,3\rangle = \frac{1}{2}\left(\langle 3,2,5\rangle - \langle -1,0,-1\rangle\right). \qquad (**)$$

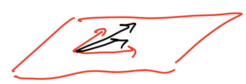
Now, (**) implies that $\boxed{\text{span}(B) \subseteq \text{span}(A)}$. Therefore $\text{span}(A) = \text{span}(B)$.

In mathematics, a common strategy to show $X = Y$ is to first show that $X \subseteq Y$, then show that $Y \subseteq X$.

You can also say that the 4 vectors are coplanar. The fact that each pair is non-parallel implies that $\text{span}(A) = \text{span}(B)$.

4. From the previous problem,

$$\langle 3,2,5\rangle = \langle 1,1,2\rangle + \langle 2,1,3\rangle.$$



The three vectors are coplanar.

Hence S has dimension 2. One basis is given by $\{\langle 1,1,2\rangle, \langle 2,1,3\rangle\}$.

But we want vectors other than $\langle 1,1,2\rangle$, $\langle 2,1,3\rangle$.

We can take

$$\langle 1,1,2\rangle + 2\langle 2,1,3\rangle = \langle 1,1,2\rangle + \langle 4,2,6\rangle = \langle 5,3,8\rangle$$
$$\langle 1,1,2\rangle - 2\langle 2,1,3\rangle = \langle 1,1,2\rangle - \langle 4,2,6\rangle = \langle -3,-1,-4\rangle$$

$\{\langle 5,3,8\rangle, \langle -3,-1,-4\rangle\}$ is another basis.

5. Linear Algebra 2.2.3 (b).

Find coefficients a, b, c such that

$$\langle 0,0,-2\rangle = a\langle 2,0,1\rangle + b\langle 1,2,-1\rangle + c\langle 3,2,2\rangle$$

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 0 & 2 & 2 & 0 \\ 1 & -1 & 2 & -2 \end{array}\right] \xrightarrow[\substack{R_2 \leftrightarrow R_3 \\ ②}]{\substack{① \\ R_1 \leftrightarrow R_3}} \left[\begin{array}{ccc|c} 1 & -1 & 2 & -2 \\ 2 & 1 & 3 & 0 \\ 0 & 2 & 2 & 0 \end{array}\right]$$

$$\xrightarrow[R_3 \to \frac{1}{2}R_3]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 2 & -2 \\ 0 & 3 & -1 & 4 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 1 & -1 & 2 & -2 \\ 0 & 1 & 1 & 0 \\ 0 & 3 & -1 & 4 \end{array}\right] \xrightarrow[R_3 \to R_3 - 3R_2]{} \left[\begin{array}{ccc|c} 1 & -1 & 2 & -2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -4 & 4 \end{array}\right]$$

$$\xrightarrow[R_3 \to -\frac{1}{4}R_3]{} \left[\begin{array}{ccc|c} 1 & -1 & 2 & -2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$c = -1$

$b + c = 0 \quad b = -c = 1$

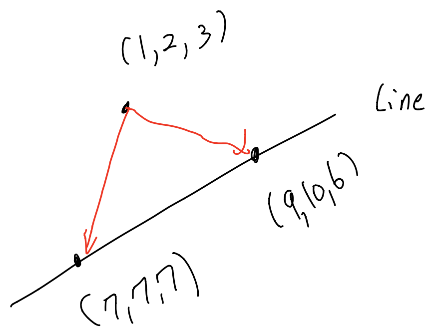
$a - b + 2c = -2$

$a = -2 + b - 2c$

$= -2 + 1 - 2(-1) = 1$

The coord $= (1, 1, -1)$

6 Linear Algebra 2.3.8 (f).



First, let's pick two points on the line.

$t=1$: $x = 5+2 = 7$, $y = 4+3 = 7$, $z = 8-1 = 7$

$t=2$: $x = 5+4 = 9$, $y = 4+3\cdot(2) = 10$, $z = 8-2 = 6$

Find two red vectors:

$\langle 7-1,\ 7-2,\ 7-3\rangle = \langle 6, 5, 4\rangle$ and $\langle 9-1,\ 10-2,\ 6-3\rangle = \langle 8, 8, 3\rangle$

clearly, they are <u>not</u> parallel.

A vector eq of the plane is $\vec{r}(s,t) = \langle 1,2,3\rangle + s\langle 6,5,4\rangle + t\langle 8,8,3\rangle$

* Sol to this question is not unique!! There are other ways to obtain the equation of the plane.